A New Approach to the Inverse Littlewood-Offord Problem

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Let \( \eta_1, \ldots, \eta_n \) be iid Bernoulli random variables, taking values \( 1, -1 \) with probability \( 1/2 \). Given a multiset \( V \) of \( n \) integers \( v_1, \ldots, v_n \), we define the concentration probability as

\[
\rho(V) := \sup_x P(v_1\eta_1 + \cdots + v_n\eta_n = x).
\]

A classical result of Littlewood-Offord and Erdos from the 1940s asserts that if the \( v_i \) are non-zero, then \( \rho(V) \) is \( O(n^{-1/2}) \). Since then, many researchers obtained improved bounds by assuming various extra restrictions on \( V \).

About five years ago, motivated by problems concerning random matrices, Tao and Vu introduced the Inverse Littlewood-Offord problem. In the inverse problem, one would like to give a characterization of the set \( V \), given that \( \rho(V) \) is relatively large.

In this talk I will present a new, general method to attack the inverse problem. As an application, we strengthen a previous result of Tao and Vu, obtaining an optimal characterization for \( V \). This immediately implies several classical theorems, such as those of Sarkozy-Szemeredi and Halasz. As another application, we obtain an asymptotic, stable version of a famous theorem of Stanley that shows that under the assumption that the \( v_i \) are different, \( \rho(V) \) attains its maximum value when \( V \) is a symmetric arithmetic progression.

All results extend to the general case when \( V \) is a subset of an abelian torsion-free group and \( \eta_i \) are independent variables satisfying some weak conditions. Inverse results in continuous setting will also be included.