Week 1 Beginning Course

Tangent Vectors and Twisting Planes: An Introduction to Legendrian Knot Theory

0.1 Course Description

A contact manifold is an odd-dimensional manifold equipped with some extra geometric structure. When the contact manifold is 3-dimensional, this structure distinguishes a special class of embedded circles, called *Legendrian knots*. In this course, we'll focus on the Legendrian knot theory associated to Euclidean $\mathbb{R}^3$ with the "standard" contact structure. We'll compare the classification of topological and Legendrian knots, paying particular attention to knot projections as a tool for computing invariants. After defining the classical invariants of Legendrian knots combinatorially, we'll interpret them geometrically in order to generalize these definitions to a broad range of contact manifolds.

0.2 Background

The required background is advanced calculus or elementary analysis (embeddings, parameterized curves, continuity, vector fields). While not required, familiarity with differential forms, manifolds, or topology may enhance the course.

1 Outline

1.1 Lecture 1

After briefly introducing topological knots, we'll define Legendrian knots in $\mathbb{R}^3$ in terms of a tangency condition. We'll introduce the two special projections of Legendrian knots and discuss the Legendrian Reidemeister moves.

1.2 Lecture 2

The second lecture will focus on the idea of a knot invariant, and we'll look at examples of topological and Legendrian knot invariants which can be computed from knot projections.

1.3 Lecture 3

In the third lecture we put Legendrian knots in the context of a contact structure on $\mathbb{R}^3$. We'll define a general contact structure on a 3-manifold and give a few examples. We'll show how a contact structure defines a framing on a Legendrian knot and reinterpret the Thurston-Bennequin number from this viewpoint.

1.4 Lecture 4

In order to interpret the rotation number geometrically, we'll introduce the notion of a Seifert surface for a knot. Using Seifert's algorithm to construct Seifert surfaces from front projections, we'll recover the combinatorial formula for the rotation number that was defined previously. We'll finish with a discussion of how the objects introduced in these lectures generalize to other settings.

21st Century Geometry

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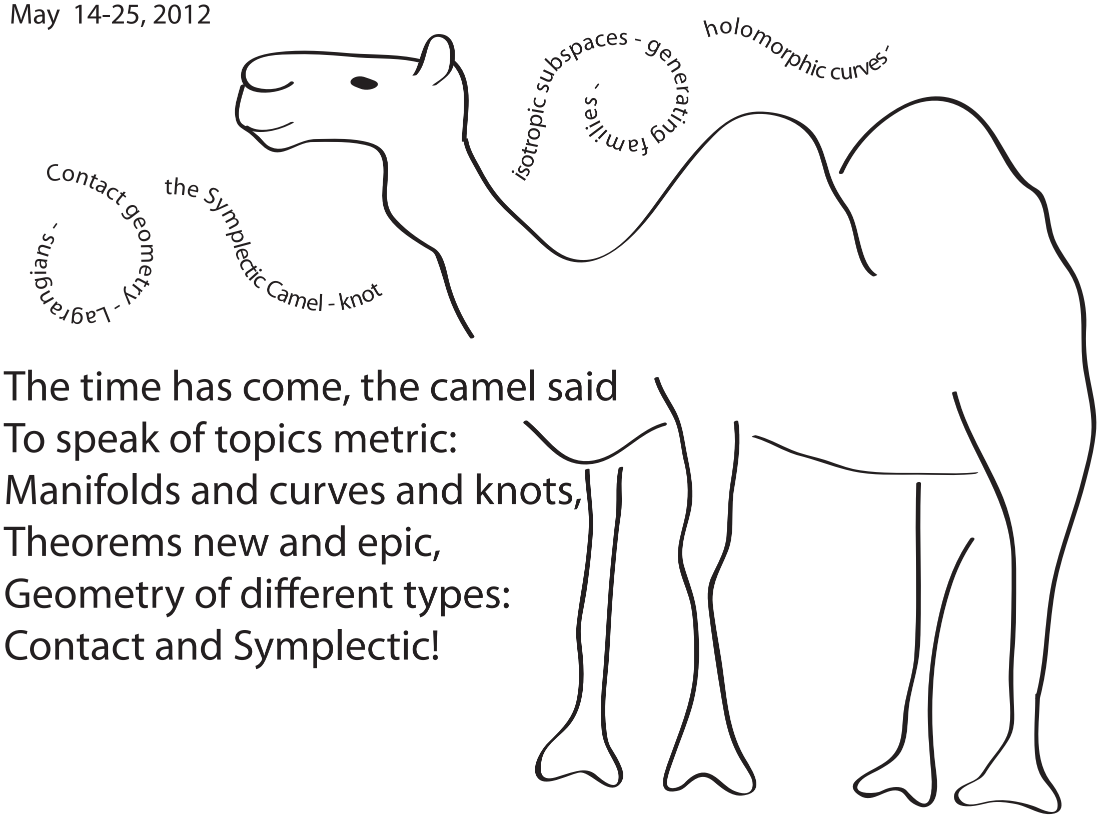

The time has come, the camel said
To speak of topics metric:
Manifolds and curves and knots,
Theorems new and epic,
Geometry of different types:
Contact and Symplectic!

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Useful Info....