Abstract: Let $I = (f_t)_{t \in [0,1]}$ be an identity isotopy on an oriented surface $M$ and denote by $\text{Cont}(I)$ the set of points $z \in M$ whose trajectory $I(z)$ is a contractible loop of $M$. A recent result of Olivier Jaulent asserts that there exists an open set $U$ whose complement is included in $\text{Cont}(I)$ and an isotopy $I = (f'_t)_{t \in [0,1]}$ on $U$ such that every trajectory $I'(z)$, $z \in U$, is homotopic to $I(z)$ in $M$ and such that the set $\text{Cont}(I') \subset U$ is empty. This implies that there exists an oriented foliation $F$ on $U$ such that every trajectory $I'(z)$ is homotopic in $U$ to a path that is transverse to $F$. This foliation $F$ seen as a singular foliation on $M$ with singularities in $M \setminus \text{Cont}(I')$ is gradient like if $M$ is a closed surface and $I$ is a Hamiltonian isotopy.

We will state two applications of this result to dynamical systems on surfaces. The first one, a joint work with Francois Beguin, Sebastiao Firmo and Tomasz Miernowski, asserts that for every $C^1$ diffeomorphism $f$ of $\mathbb{R}^2$ that extends to a $C^1$ diffeomorphism of $S^2$ and that admits an invariant finite measure not included in its fixed point set, there exists a subset $X \subset \text{Fix}(f)$ which is invariant by the centralizer of $f$ in $\text{Homeo}(\mathbb{R}^2)$. The second one, a joint work with Andres Koropecki and Meysam Nassiri, asserts that if $U$ is an open subset of a closed surface $M$ that is invariant by an area preserving homeomorphism of $M$ and if $p$ is a fixed regular end of $U$ with an irrational prime end rotation number, then the boundary $Z(p)$ is an annular set with no periodic point or a cellular set with a unique periodic (and so fixed) point.