The proof of Lemma 4.8 does not really work for the range of exponents claimed in the paper. Indeed we follow Müller's proof (see reference [13] in the paper), which works for maps $u \in W^{1,p}$ with $p \geq \frac{n^2}{n+1}$. Since Theorem 4.7 is based on a slicing technique which reduces the proof to Lemma 4.8, tthe arguments given in the paper are incomplete and give the claim only for the range of exponents covered in Müller's result. However, in a subsequence work with F. Ghiraldin, we have been able to extend Müller's Theorem to the full range of exponents claimed in my 2002 paper. Therefore, both the statements of Theorem 4.7 and Lemma 4.8 are correct. For the complete proof we refer to

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An extension of the identity **Det** = **det**.
C. R. Math. Acad. Sci. Paris 348 (2010), no. 17-18, 973–976.