The statement in Proposition 0.2 is incorrect because the starting triple $(0,0,0)$ does not satisy the inequality (12).

A starting triple does exist, but the proof is slightly more complicated. One reasonably short construction is the following. We define $p_0 = 0$, $v_0(x,t) = (e(t)(1-\delta_1))^{1/2}(\cos\bar\lambda x_3, \sin\bar\lambda x_3, 0)$ and

$$\mathring{R}_0(x,t) := \bar\lambda^{-1}\frac{d}{dt}(e(t)(1-\delta_1))^{1/2}\begin{pmatrix} 0 & 0 & \sin\bar\lambda x_3 \\ 0 & 0 & -\cos\bar\lambda x_3 \\ \sin\bar\lambda x_3 & -\cos\bar\lambda x_3 & 0 \end{pmatrix},$$

where

$$\bar\lambda := \bar C\delta_1^{-1},$$

for a constant $\bar C$ which depends only upon the function e and the parameter η.

It is straightforward to check that the triple solves the Euler-Reynolds system (5) and it is also obvious that it satisfies (12). Note that we need to show (10), (11) and (15), but also (26) (the latter would be an obvious outcome of the inductive estimates on the differences $v_q - v_{q-1}$ if the starting v_0 were 0: since now $v_0 \neq 0$, (26) must in fact be checked as well; not that there is no need to check instead (29), since p_0 is indeed 0).

- The inequality (10) is equivalent to $C\bar\lambda^{-1} \le \eta\delta_1$. where the constant C depends only upon e, which just requires $\bar C$ large enough. This is the only condition on $\bar C$, which for the remaining inequalities is considered to be fixed, and thus depending only upon e and η.
- The inequality (11) is equivalent to $C \le \delta_1\lambda_0 = a^{-b}a^{bc} = a^{b(c-1)}$, where the constant C depends only upon e and η, which just requires a large enough depending upon e, b and c.
- The inequality (15) is implied by $C \le \delta_1\delta_0^{1/2}\lambda_0 = a^{-b-1/2+bc}$. Again, since $cb - b \ge \frac{3}{2}b \ge \frac{3}{2}$, this just requires a sufficiently large choice of a.
- The first inequality in (26) is implied by (25) and $\|v_0\| \le C$, where the constant C depends only upon e (and not on η!): we just need M to be large depending on e.
- The second inequality in (26) follows from (25) if we can show that $\|v_0\|_1 \le \delta_0^{1/2}\lambda_0$. However this is again equivalent to $C \le \delta_1\delta_0^{1/2}\lambda_0$, which has been already shown above.