1. Minor errors/typos

(1) Line 7 after (3.4), Tan^α should be Tan_α.
(2) Eq. (3.12), line before and two lines after: the integration variable should be y (and not x) and the domain of the integrals should be $B_{\rho r_i}(x)$.
(3) Last two lines of Proposition 3.12: the argument shows the inclusion $\supset$, rather than $\subset$.
(4) Displayed eq. before (3.13): the definition of $E_{i,j,k}$ should ask for $x \in E$.
(5) Inequality (3.14): a factor r^α is missing.
(6) Second displayed equation after (3.14), a factor $r_i^{-\alpha}$ is missing in the first $\limsup$.
(7) First line of page 26: $\mathrm{supp}\,(r)$ is actually $\mathrm{supp}\,(\nu)$.
(8) In page 97 the integrand in the first displayed formula is missing the factor $e^{-s|z|^2}$.

Many thanks to Federico Glaudo and Simone Steinbrüchel.

2. Substantial errors

(i) The proof at page 92 of (8.21) for the case $m = 2$ is wrong: the implication suggested in the last line (namely that the existence of the vectors y and z implies $\alpha_2 \geq 1$ by simple linear algebra) is incorrect. A correct argument goes as follows. We consider the plane W spanned by y and z and from the identitities derived thus far we certainly conclude that $\mathrm{tr}\, b_2^{(1)} \llcorner W = 2$. In particular from Lemma 8.7 we conclude $\mathrm{tr}\, b_2^{(1)} \llcorner W^\perp = 0$. We can now use the identity (8.22) (which is valid for any m) to derive that λ is supported in the plane W, which in turn implies $\lambda = \mathcal{H}^2 \llcorner W$. This is already the desired conclusion to achieve Proposition 8.5, rather than (8.21). However (8.21) follows as well.

Many thanks to Federico Glaudo for spotting the error and suggesting the alternative argument!

(ii) Federico has also pointed out to me that I missed the following point: the product of two uniform measures is a uniform measure as well. For this reason Question 10.19 in the book has the simple answer that, if $C \times C \subset \mathbb{R}^8$ is the product of two light cones, then $\mathcal{H}^6 \llcorner C \times C$ is indeed a uniform measure. The question should have therefore been stated in the following way:

(Q) Are there k-uniform measures which are not the product of light cones and flat measures?

Recently Dali Nimer has classified all three-dimensional cones $Z \subset \mathbb{R}^n$ for which $\mathcal{H}^3 \llcorner Z$ is a uniform measure, showing that there are indeed infinitely many. Dali's work provides thus a positive answer even to the above adjusted version of Question 10.19. The interested reader may find Dali's work here:

https://arxiv.org/abs/1608.02604

I have not been able to find a reference in the literature to Federico's remark that the product of two uniform measures is a uniform measure, although it was known to Preiss. I attach Federico's proof for the reader's convenience.

Product of Uniform Measure is Uniform

We will denote with $B^n(x,r)$ the open ball in $\mathbb{R}^n$ with center x and radius r.

Definition 1 (k-uniform measure)**.** A measure μ on the Borel sets of $\mathbb{R}^n$ is k-uniform if $0 \in \operatorname{supp}(\mu)$ and for any $x \in \operatorname{supp}(\mu)$ and $r > 0$ it holds

$$\mu(B^n(x,r)) = \mathcal{L}^k(B^k(0,r)) .$$

The set of k-uniform measures will be denoted by $\mathcal{U}^k(\mathbb{R}^n)$.

Lemma 2. *Let $\varphi : \mathbb{R} \to \mathbb{R}^+$ be a Borel function, μ a k-uniform measure and y a point in the support of μ. Then*

$$\int_{\mathbb{R}^n} \varphi(|x-y|)\,\mathrm{d}\mu(x) = \int_{\mathbb{R}^k} \varphi(|z|)\,\mathrm{d}\mathcal{L}^k(z) .$$

Proof. It can be find in [1, pag. 70 – Lemma 7.2]. □

Proposition 3. *Let $\mu \in \mathcal{U}^h(\mathbb{R}^n)$ and $\nu \in \mathcal{U}^k(\mathbb{R}^m)$ be two uniform measures. Then the product measure $\mu \otimes \nu$ is a $(h+k)$-uniform measure in $\mathbb{R}^{n+m}$.*

Proof. First of all it holds that $\operatorname{supp}(\mu \otimes \nu) = \operatorname{supp}(\mu) \times \operatorname{supp}(\nu)$ as this is true in general. Therefore we have $(0,0) \in \operatorname{supp}(\mu \otimes \nu)$.

In order to check the uniformity property for $\mu \otimes \nu$ let us fix $(\bar{x}, \bar{y}) \in \operatorname{supp}(\mu \otimes \nu)$. We know that $\bar{x} \in \operatorname{supp}(\mu)$ and $\bar{y} \in \operatorname{supp}(\nu)$. Applying Fubini-Tonelli we get

$$\mu \otimes \nu \left(B^{n+m}((\bar{x},\bar{y}),r)\right) = \int_{B^n(\bar{x},r)} \nu\left(B^m\left(\bar{y}, \sqrt{r^2-(x-\bar{x})^2}\right)\right) \mathrm{d}\mu(x)$$

and thanks to Lemma 2 we can continue our chain of equalities

$$\begin{aligned} &= \int_{B^n(\bar{x},r)} \mathcal{L}^k\left(B^k\left(0, \sqrt{r^2-(x-\bar{x})^2}\right)\right) \mathrm{d}\mu(x) = \int_{B^h(0,r)} \mathcal{L}^k\left(B^k\left(0, \sqrt{r^2-z^2}\right)\right) \mathrm{d}\mathcal{L}^h(z) \\ &= \mathcal{L}^h \otimes \mathcal{L}^k\left(B^{h+k}(0,r)\right) = \mathcal{L}^{h+k}\left(B^{h+k}(0,r)\right) \end{aligned}$$

and this is exactly the uniformity condition we were looking for. □

References

[1] Camillo De Lellis, *Rectifiable Sets, Densities and Tangent Measures*, Zurich lectures in advanced mathematics, European Mathematical Society.