In the paper we do not give a proof of equation (18), a key statement of Theorem 4.2. The statement is not at all trivial and requires indeed a certain amount of work.

Proof of (18). Let $\{x_i\}$ be the set such that $\mathcal{L}^3(F(x_i)) > 0$.

Step 1 First of all we show that each $F(x_i)$ is a Caccioppoli set. Set $x = x_i$, let $r \in R_x$ and consider the Caccioppoli set $C = \mathrm{im}_T(u, B(x,r))$. For $\mathcal{L}^3$–a.e. y, $u(y)$ is a point of density 1 for $\mathrm{im}_G(u,\Omega)$. A standard Fubini argument implies that for $\mathcal{L}^1$–a.e. r we have the property that $\mathcal{H}^2$–a.e. point $y \in u(\partial B(x,r))$ is of density 1 for $\mathrm{im}_G(u,\Omega)$. Assume that r enjoys this property. $D := \mathrm{im}_T(u, B(x,r)) \setminus \mathrm{im}_G(u,\Omega)$ is a Caccioppoli set and $\partial^* D \subset \partial^* \mathrm{im}_T(u, B(x,r)) \cup \partial^* \mathrm{im}_G(u,\Omega)$. By Lemma 3.10, $\mathcal{H}^2(\partial^* \mathrm{im}_T(u, B(x,r)) \setminus u(\partial B(x,r))) = 0$. Thus, since $\mathcal{H}^2$ a.e. $y \in u(\partial B(x,r))$ is of density 1 for $\mathrm{im}_G(u,\Omega)$, we conclude that, up to $\mathcal{H}^2$-null sets, $\partial^* D \subset \partial^* \mathrm{im}_G(u,\Omega)$.

Consider now a sequence of radii $r_j \downarrow 0$ as above and the corresponding sets $D_j := \mathrm{im}_T(u, B(x,r_j)) \setminus \mathrm{im}_G(u,\Omega)$. By the monotonicity property of Lemma 3.12(ii), $\bigcap D_j = F(x)$. On the other hand, $\mathrm{Per}\, D_j \leq \mathrm{Per}\, \mathrm{im}_G(u,\Omega) = 0$. Thus we conclude that $F(x)$ is a Caccioppoli set and that

$$\mathrm{Per}\,(F(x)) \leq \mathrm{Per}\, \mathrm{im}_G(u,\Omega)\,. \tag{1}$$

Step 2 Consider a finite number of points $x_1, \ldots, x_N$. We can argue as above and define the set $D_j := (\cup_{i=1}^N \mathrm{im}_T(u, B(x_i, r_j))) \setminus \mathrm{im}_G(u,\Omega)$. Choosing suitable radii $r_j \downarrow 0$ we then conclude that

$$\mathrm{Per}\,\left(\cup_{i=1}^N F(x_i)\right) \leq \mathrm{Per}\, \mathrm{im}_G(u,\Omega)\,.$$

We next claim that $\mathcal{H}^2(\partial^* F(x_i) \cap \partial^* F(x_j)) = 0$ if $i \neq j$. Consider indeed a radius $r \in R_{x_i} \cap R_{x_j}$ with $r < \|x_i - x_j\|$. By Lemma 3.12, $\mathrm{im}_T(u, B(x_i,r)) \cap \mathrm{im}_T(u, B(x_j,r)) = \emptyset$. Moreover,

$$\begin{aligned} & \mathcal{H}^2(\partial^* \mathrm{im}_T(u, B(x_i,r)) \setminus u(\partial B(x_i,r))) \\ = \; & \mathcal{H}^2(\partial^* \mathrm{im}_T(u, B(x_j,r)) \setminus u(\partial B(x_i,r))) = 0. \end{aligned}$$

Recalling that u is injective on Ω_d and that Definition 3.11(ii) holds, if we choose the radius r appropriately we conclude that

$$\mathcal{H}^2(\partial^* \mathrm{im}_T(u, B(x_i,r)) \cap \partial^* \mathrm{im}_T(u, B(x_j,r))) = 0\,.$$

Since $F(x_i) \subset \mathrm{im}_T(u, B(x_i,r))$, we must have

$$\partial^* F(x_i) \subset \mathrm{im}_T(u, B(x_i,r)) \cup \partial^* \mathrm{im}_T(u, B(x_i,r))\,.$$

Thus, we conclude that $\mathcal{H}^2(\partial^* F(x_i) \cap \partial^* F(x_j)) = 0$. But then, we have

$$\sum_{i=1}^{N} \operatorname{Per} F(x_i) = \operatorname{Per} \left(\cup_{i=1}^{N} F(x_i) \right) \leq \operatorname{Per} \operatorname{im}_G(u, \Omega) \,. \tag{2}$$

Letting $N \uparrow \infty$ we conclude the desired inequality. □

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