In the paper we do not give a proof of equation (18), a key statement of Theorem 4.2. The statement is not at all trivial and requires indeed a certain amount of work.

Proof of (18). Let \( \{x_i\} \) be the set such that \( \mathcal{L}^3(F(x_i)) > 0 \).

**Step 1** First of all we show that each \( F(x_i) \) is a Caccioppoli set. Set \( x = x_i \), let \( r \in R_x \) and consider the Caccioppoli set \( C = \text{im}_T(u, B(x, r)) \).

For \( \mathcal{L}^3 \)-a.e. \( y, u(y) \) is a point of density 1 for \( \text{im}_G(u, \Omega) \). A standard Fubini argument implies that for \( \mathcal{L}^1 \)-a.e. \( r \) we have the property that \( \mathcal{H}^2 \)-a.e. point \( y \in u(\partial B(x, r)) \) is of density 1 for \( \text{im}_G(u, \Omega) \). Assume that \( r \) enjoys this property. \( D := \text{im}_T(u, B(x, r)) \setminus \text{im}_G(u, \Omega) \) is a Caccioppoli set and \( \partial^* D \subset \partial^* \text{im}_T(u, B(x, r)) \cup \partial^* \text{im}_G(u, \Omega) \).

By Lemma 3.10, \( \mathcal{H}^2(\partial^* \text{im}_T(u, B(x, r)) \setminus u(\partial B(x, r))) = 0 \). Thus, since \( \mathcal{H}^2 \) a.e. \( y \in u(\partial B(x, r)) \) is of density 1 for \( \text{im}_G(u, \Omega) \), we conclude that, up to \( \mathcal{H}^2 \)-null sets, \( \partial^* D \subset \partial^* \text{im}_G(u, \Omega) \).

Consider now a sequence of radii \( r_j \downarrow 0 \) as above and the corresponding sets \( D_j := \text{im}_T(u, B(x, r_j)) \setminus \text{im}_G(u, \Omega) \). By the monotonicity property of Lemma 3.12(ii), \( \bigcap D_j = F(x) \).

On the other hand, \( \text{Per} \ D_j \leq \text{Per} \ \text{im}_G(u, \Omega) = 0 \). Thus we conclude that \( F(x) \) is a Caccioppoli set and that

\[
\text{Per} \ (F(x)) \leq \text{Per} \ \text{im}_G(u, \Omega).
\] (1)

**Step 2** Consider a finite number of points \( x_1, \ldots, x_N \). We can argue as above and define the set \( D_j := (\bigcup_{i=1}^N \text{im}_T(u, B(x_i, r_j))) \setminus \text{im}_G(u, \Omega) \).

Choosing suitable radii \( r_j \downarrow 0 \) we then conclude that

\[
\text{Per} \ (\bigcup_{i=1}^N F(x_i)) \leq \text{Per} \ \text{im}_G(u, \Omega).
\]

We next claim that \( \mathcal{H}^2(\partial^* F(x_i) \cap \partial^* F(x_j)) = 0 \) if \( i \neq j \). Consider indeed a radius \( r \in R_{x_i} \cap R_{x_j} \) with \( r < \|x_i - x_j\| \). By Lemma 3.12, \( \text{im}_T(u, B(x_i, r)) \cap \text{im}_T(u, B(x_j, r)) = \emptyset \). Moreover,

\[
\mathcal{H}^2(\partial^* \text{im}_T(u, B(x_i, r)) \setminus u(\partial B(x_i, r))) = \mathcal{H}^2(\partial^* \text{im}_T(u, B(x_j, r)) \setminus u(\partial B(x_j, r))) = 0.
\]

Recalling that \( u \) is injective on \( \Omega_d \) and that Definition 3.11(ii) holds, if we choose the radius \( r \) appropriately we conclude that

\[
\mathcal{H}^2(\partial^* \text{im}_T(u, B(x_i, r)) \cap \partial^* \text{im}_T(u, B(x_j, r))) = 0.
\]

Since \( F(x_i) \subset \text{im}_T(u, B(x_i, r)) \), we must have

\[
\partial^* F(x_i) \subset \text{im}_T(u, B(x_i, r)) \cup \partial^* \text{im}_T(u, B(x_i, r)).
\]
Thus, we conclude that $H^2(\partial^* F(x_i) \cap \partial^* F(x_j)) = 0$. But then, we have
\[ \sum_{i=1}^{N} \text{Per} F(x_i) = \text{Per} \left( \bigcup_{i=1}^{N} F(x_i) \right) \leq \text{Per} \text{im}_G(u, \Omega). \] (2)
Letting $N \uparrow \infty$ we conclude the desired inequality.

Acknowledgments I am very grateful to Duvan Henao for pointing out the incompleteness of the proof of Theorem 4.2.