The argument given for Proposition 4.1 is incorrect. In particular, after equation (25) we claim that, since \( h''_{t,x} \) changes sign at most once in \([u^-, u^+]\), \( h_{t,x}(u^-) = h_{t,x}(u^+) = 0 \) and \( h_{t,x} \geq 0 \) on \([u^-, u^+]\), then \( h''_{t,x} \leq 0 \). This is, however, an obviously false claim. I do not know whether Proposition 4.1 holds in the generality claimed by the paper. Therefore I do not know whether conclusion (c) holds in the generality claimed by Theorem 1.1. However, I can supply a proof of Proposition 4.1 (and hence of Theorem 1.1(c)) under the following stronger assumption:

(A) \( f'' \) does not vanish at infinite order, that is, if \( f''(x_0) = 0 \), then there are \( c_0 \neq 0 \) and \( j \in \mathbb{N} \) such that \( f''(x) = c_0(x - x_0)^j + o(x - x_0)^j \) in a neighborhood of \( x_0 \).

Condition (A) is obviously fulfilled by an analytic function. Variations of the argument given below lead to weaker conditions then (A).

**Proof.** Following the arguments of Proposition 4.1 till (25), what we need to show is the following. Fix a point \( x_0 \) with \( f''(x_0) = 0 \). Then there are constants \( C \) and \( \varepsilon \) with the following property: if there is an admissible shock wave with \( a, b \) as traces such that

- \( b - a < \varepsilon \);
- \( a < x_0 < b \);

then

\[
\int_a^b |f''(x)| \leq C|f'(b) - f'(a)|.
\]

The condition of admissibility of the shock is that \( f \) lies above the line connecting \((a, f(a))\) and \((b, f(b))\), in case \( b \) is the right and \( a \) the left trace, and that \( f \) lies below otherwise.

We can therefore assume, without loss of generality, that

- \( x_0 = 0 \);
- \( f(0) = f'(0) = f''(0) = 0 \).

From (A) we know that \( f(x) = c_0 x^n + o(x^n) \) with \( c_0 \neq 0 \). If \( n \) is even, the function \( f \) is convex (or concave) in a neighborhood of 0 and the proof of (1) is obvious. We therefore assume \( n = 2k+1 \) with \( k \geq 1 \) and \( c_0 = \pm 1 \). The case \( c_0 = -1 \) can be handled similarly and we therefore assume \( c_0 = 1 \). Analogously we assume \( a < 0 < b \). The inequality (1) reduces then to

\[
f'(b) + f'(a) \leq C(f'(b) - f'(a))
\]

which in turn is equivalent to the existence of a \( \delta > 0 \) such that

\[
f'(b) \leq (1 - \delta)f'(a).
\]

Note that \( f'' > 0 \) on some interval \([0, \alpha]\), and hence it suffices to prove the inequality (3) for the largest possible \( b \) in a neighborhood of 0 which
can be connected to \( a \) by an admisible shock. This is achieved when the line \( r \) passing through \((a, f(a))\) and \((b, f(b))\) is tangent to the graph of \( f \) in \((b, f(b))\). In this case we have
\[
f'(b) = \frac{f(b) - f(a)}{b - a}.
\] (4)

The inequality (3) becomes then
\[
f(b) - f(a) \leq (1 - \delta)f'(a)(b - a).
\] (5)

Recalling the assumption (A), for any fixed \( \eta > 0 \), there is an \( \varepsilon > 0 \) such that (4) implies
\[
(2k + 1)(1 - \eta)b^{2k} \leq \frac{(1 + \eta)b^{2k+1} + (1 + \eta)|a|^{2k+1}}{b + |a|} \leq \frac{(1 + \eta)(b^{2k+1} + |a|^{2k+1})}{b}.
\]

This in turn gives
\[
\frac{2k - (2k + 2)\eta b^{2k+1}}{1 + \eta} \leq |a|^{2k+1}.
\]

Choosing \( \eta \leq \frac{2}{3} \), we conclude \( |a| \geq b \).

Using this last information and (A), we can estimate
\[
f(b) - f(a) \leq (1 + \eta)(b^{2k+1} + |a|^{2k+1}) \leq 2(1 + \eta)|a|^{2k+1}
\]
and
\[
f'(a)(b - a) \geq (2k + 1)(1 - \eta)|a|^{2k}(b + |a|) \geq 3(1 - \eta)|a|^{2k+1}.
\]

Choosing \( \eta = \frac{1}{7} \) and \( \delta = \frac{1}{9} \) we get inequality (5). \( \Box \)

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