The proof of Theorem 3.22 contains a mistake in the existence of the regular Lagrangian flow Φ. Indeed, in the step about the stronc convergence of Φ_n, the curve $t \mapsto \Gamma^j(t,x)$ does not solve any ODE and hence it is not clear why it should be Lipschitz. I give here an alternative argument.

Step 1 We use Lemma 3.7 to define the solutions of the continuity equations at any time.

Step 2 We strenghten Corollary 3.19: The weak* convergence of $\zeta_n(t,\cdot)$ to $\zeta(t,\cdot)$ holds at every time t. Indeed let φ be a test functions which depends only on the space variable x and set

$$f_n(t) := \int \zeta_n(t,x)\varphi(x)dx$$

$$f(t) := \int \zeta(t,x)\varphi(x)dx$$

f_n and f are continuous by Lemma 3.7. Using the equations defininig them you get also

$$f_n'(t) = \int \zeta_n(t,x)\nabla\varphi(x)\cdot b_n(t,x)dx\,.$$

Thus $\|f_n'\|_{C^0} \leq C$ for some constant C which depends only on φ.

By Ascoli-Arzelà f_n converges uniformly to some continuous function. However, by the weak* convergence of ζ_n to ζ (in time *and* space), $f_n \to f$ uniformly. Ne concludi che f_n converges to f uniformly, and hence pointwise for every t. Since $\|\zeta_n(t,\cdot)\|_\infty$ is uniformly bounded and φ is an arbitrary test function, we conclude that $\zeta_n(t,\cdot) \to \zeta(t,\cdot)$ weak* in L^∞ for every t.

Step 3 We next strenghten Corollary 3.20, claiming that $u_n(t,\cdot)$ converges strongly in L^1_{loc} to $u(t,\cdot)$ for all t. Indeed note that Step 2, the renormalization property and Corollary 3.14 imply that

$$\zeta_n(t,\cdot)u_n^2(t,\cdot) \to \zeta(t,\cdot)u^2(t,\cdot) \quad \text{and} \quad \zeta_n(t,\cdot)u_n(t,\cdot) \to \zeta(t,\cdot)u(t,\cdot)$$

weakly* in L^∞ for every t.

Step 4 We now get back to the existence part in the proof of Theorem 3.22. In *Existence. Step 2: Stromg convergence*: we can apply the Step 3 above to the maps w_n and conclude that $w_n(t,\cdot)$ converges strongly to $w(t,\cdot)$ for all t. Having obtained this property, we can continue with the rest of the proof, which is correct.

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