Theorem 1 in the paper is quoted incorrectly from Theorem 1.1 in the reference

[2] De Lellis, C., Müller, S.: Optimal rigidity estimates for nearly umbilical surfaces. J. Differential Geom. 69, 75–110 (2005).

In fact in the second part of the statement the assumption in [2, Theorem 1.1] is $\int_{\Sigma} |\mathring{A}|^2 \leq 8\pi$ and not $\int_{\Sigma} |\mathring{A}|^2 \leq 4\pi$. Note that the assumption is instead correctly reported to be $\int_{\Sigma} |\mathring{A}|^2 \leq 8\pi$ in Theorem 2.

The mistake in Theorem 1 is a funny coincidence, because I was made aware by Ruben Jakob that the proof given in [2] of Theorem 1 has a trivial computational mistake (more precisely a factor 2 missing on the left hand side of [2, Equation (7)]): after adjusting the rest of the argument to take into account this factor, we are able to conclude the second part of Theorem 1 with any constant C_0 smaller than 8π in $\int_{\Sigma} |\mathring{A}|^2 \leq C_0$. On the other hand that argument fails precisely at $C_0 = 8\pi$. We did not realize this at the time: the replacement of 8π with 4π in Theorem 1 is just a mere accident and not an attempt at correcting the error contained in [2]. Moreover, as explained in the errata for [2] (that the reader can find on my website at

Errata-for-reference-[2])

the statement is still correct with $C_0 = 8\pi$ (and in fact with any $C_0 < 4\pi^2$) but the argument needs much more sophisticated results than the very elementary ones used in [2].

Many thanks to Ruben Jakob for pointing out the inconsistencies.