Equation (52) of Proposition 5.2 in the journal version (equation (5.6) in the arXiv version) must be substituted with

$$\int_0^{2\pi} v_\phi(\phi,\sigma)\sin\frac{\phi}{2}\,d\phi = 0\,. \tag{0.1}$$

We will first show where the (computational) mistake in the proof of Proposition 5.2 is, leading to this new identity and we will then show how to modify the remaining parts of the paper where the incorrect equation (52) is used.

0.1. **Proof of** (0.1)**.** The displayed equation before (56) (resp. (5.10) in the arXiv version) is correct and reads

$$\int_{\partial B_{r_0}\setminus\{p_j\}} \left(|\nabla w_j(q)|^2 \nu(q)\cdot\tau(p_j) - 2\frac{\partial w_j}{\partial \tau}(q)\nabla w_j(q)\cdot(\nu(q)-\nu(p_j))\right) d\mathcal{H}^1(q) = 0\,. \tag{0.2}$$

As in the paper we assume without loss of generality that p_j lies on the horizontal axis and introduce polar coordinates. In particular

$$\tau(p_j) = (0,1)$$
$$\nu(p_j) = (1,0)$$
$$\nu = (\cos\phi, \sin\phi)$$

and

$$|\nabla w_j|^2 = \frac{1}{r_0}^{2} w_{j,\phi}^2 + w_{j,r}^2$$
$$\frac{\partial w_j}{\partial \tau} = \frac{1}{r_0} w_{j,\phi}$$
$$\nabla w_j\cdot(\nu-\nu(p_j)) = w_{j,\phi} - w_{j,r}\cos\phi + \frac{\sin\phi}{r_0} w_{r,\phi}\,.$$

Inserting the latter inside (0.2) we achieve

$$r_0\int_0^{2\pi}\left(w_{j,r}^2 + \frac{1}{r_0^2}w_{j,\phi}^2\right)(\phi, r_0)\sin\phi, d\phi$$
$$-2\int_0^{2\pi} w_{j,\phi}(\phi,r_0)(w_{j,r}(\phi,r_0)(1-\cos\phi) + r_0^{-1}w_{j,\phi}\sin\phi)\,d\phi = 0\,.$$

In particular equation (56) of the printed paper (resp. equation (5.10) in the arXiv version) must be substituted with

$$\underbrace{\int_0^{2\pi}\left(r_0 w_{j,r}^2 - \frac{1}{r_0}w_{j,\phi}^2\right)(\phi,r_0)\sin\phi\, r_0\,d\phi}_{=:A_j} - \underbrace{2\int_0^{2\pi}(w_{j,r}w_{j,\phi})(\phi,r_0)(1-\cos\phi)\,d\phi}_{=:B_j} = 0 \tag{0.3}$$

In turn (59) (resp. (5.13)) remains the same, namely

$$\lim_{j\to\infty}\delta_j^{-1}A_j$$
$$= \sqrt{\frac{2}{\pi}}\int_0^{2\pi}\left(\sin\frac{\phi}{2}\left(\frac{\dot\lambda(\sigma)}{\sqrt{2\pi}}\cos\frac{\phi}{2} + \frac{v(\phi,\sigma)}{2} - v_t(\phi,\sigma)\right) - \cos\frac{\phi}{2}v_\phi(\phi,\sigma)\right)\sin\phi\,d\phi\,, \tag{0.4}$$

while (60) (resp. (5.14)) becomes

$$
\begin{aligned}
&\lim_{j\to\infty} \delta_j^{-1} B_j \\
=&\sqrt{\frac{2}{\pi}} \int_0^{2\pi} \left(\sin\frac{\phi}{2} v_\phi(\phi,\sigma) + \cos\frac{\phi}{2} \left(\frac{\dot\lambda(\sigma)}{\sqrt{2\pi}} \cos\frac{\phi}{2} + \frac{v(\phi,\sigma)}{2} - v_t(\phi,\sigma) \right) \right) (1-\cos\phi)\, d\phi\,. \quad (0.5)
\end{aligned}
$$

In particular, since $A_j - B_j = 0$, we infer

$$
\begin{aligned}
0 =& \frac{\dot\lambda(\sigma)}{\pi} \int_0^{2\pi} \left(\sin\frac{\phi}{2}\cos\frac{\phi}{2}\cos\phi - \cos^2\frac{\phi}{2}(1-\cos\phi) \right) d\phi \\
&+ \sqrt{\frac{2}{\pi}} \int_0^{2\pi} \left(\frac{v}{2} - v_t \right)(\phi,\sigma) \left(\sin\frac{\phi}{2}\sin\phi - \cos\frac{\phi}{2}(1-\cos\phi) \right) d\phi \\
&+ \sqrt{\frac{\pi}{2}} \int_0^{2\pi} v_\phi(\phi,\sigma) \left(-\cos\frac{\phi}{2}\sin\phi - \sin\frac{\phi}{2}(1-\cos\phi) \right) d\phi \qquad (0.6)
\end{aligned}
$$

Observe next that

$$
\begin{aligned}
&\sin\frac{\phi}{2}\cos\frac{\phi}{2}\cos\phi - \cos^2\frac{\phi}{2}(1-\cos\phi) = \frac{1}{2}\sin^2\phi - \frac{1}{2}(1+\cos\phi)(1-\cos\phi) = 0\,, \\
&\sin\frac{\phi}{2}\sin\phi - \cos\frac{\phi}{2}(1-\cos\phi) = \left(\cos\frac{\phi}{2}\cos\phi + \sin\frac{\phi}{2}\sin\phi \right) - \cos\frac{\phi}{2} \\
=& \cos\left(\phi - \frac{\phi}{2}\right) - \cos\frac{\phi}{2} = 0 \\
&-\cos\frac{\phi}{2}\sin\phi - \sin\frac{\phi}{2}(1-\cos\phi) = -\left(\sin\phi\cos\frac{\phi}{2} - \cos\phi\sin\frac{\phi}{2} \right) - \sin\frac{\phi}{2} \\
=& -\sin\left(\phi - \frac{\phi}{2}\right) - \sin\frac{\phi}{2} = -2\sin\frac{\phi}{2}
\end{aligned}
$$

Inserting the latter in (0.6) we conclude (0.1).

0.2. **How to use** (0.1) **in the remaining arguments.** Note that (52) (resp. (5.6)) is used only in the proof of Proposition 7.2 to derive (88) (resp. (7.8)). The latter must be now substituted with

$$
a_0(t) \underbrace{\int_0^{2\pi} \left(\sin\frac{\phi}{2} + \frac{1}{2}\cos\frac{\phi}{2}(\phi - \pi) \right) \sin\frac{\phi}{2}\, d\phi}_{=:I} = 0\,.
$$

We compute the integral I as

$$
\begin{aligned}
I =& \int_0^{2\pi} \left(\sin^2\frac{\phi}{2} + \frac{1}{2}\cos\frac{\phi}{2}\sin\frac{\phi}{2}(\phi-\pi) \right) d\phi \\
=& \int_0^{2\pi} \left(\frac{1}{2}(1-\cos\phi) + \frac{\phi-\pi}{4}\sin\phi \right) d\phi \\
=& \pi - \frac{\phi-\pi}{4}\cos\phi\Big|_0^{2\pi} + \frac{1}{4}\int_0^{2\pi} \cos\phi\, d\phi = \frac{\pi}{2}\,.
\end{aligned}
$$

So we actually infer $a_0(t) = 0$, which in turn implies $\lambda(t) = d(e^t - 1)$. We can now insert the latter inside (85) (resp. (7.5)) to get

$$1 = 2d^2 \int_0^1 e^{2t}\, dt \geq 2d^2 \max\left\{(1-\eta)\int_0^1 e^{2t}\, dt, (1+\eta)^{-1}\int_1^2 e^{2t}\, dt\right\},$$

which is equivalent to

$$e^4 - e^2 \geq \max\{(1-\eta)(e^2-1), (1+\eta)^{-1}(e^6 - e^4)\},$$

which in turn is equivalent to

$$e^2 \geq \max\{(1-\eta), (1+\eta)^{-1}e^4\}.$$

Since $0 < \eta < 1$, the latter would imply $e^2 \geq \frac{e^4}{2}$, which is clearly a contradiction.