In the first line of the proof of Lemma 5.2 we are actually using $4\delta_{q+2} \leq \delta_{q+1}\lambda_q^{-\alpha}$ (which is also correct).

Moreover the last inequality in (6.9) is not quite right and should rather be

$$\lesssim \frac{\delta_{q+1}^{1/2}\delta_q^{1/2}\lambda_q}{\lambda_{q+1}^{1-\alpha}\lambda_q^{-3\alpha/2}} \lesssim \frac{\delta_{q+1}^{1/2}\delta_q^{1/2}\lambda_q}{\lambda_{q+1}^{1-3\alpha}} \,.$$

Since our goal is to prove (6.1), the latter inequality still serves our purposes.

Thanks to Manh Khang Huynh for pointing out these errors.