

Errata to Allard's Interior Regularity Theorem: an invitation to stationary varifolds

Many thanks to Mohsen Kalthori for pointing out the following mistakes in the notes:

(1) The statement of Theorem 3.1 is missing the important assumption that $x_0 \in \text{spt}(\mu_V)$. This assumption is implicitly used in the arguments given in the notes, and it is also necessary, otherwise one can simply give counterexamples to the statement.

(2) Page 10 line 8, $\sum_{i=1}^k$ should be removed after the second equality sign.

(3) The proof given for Proposition 5.1 should be amended (see comment (5) below) and the change would give a smaller radius $r/64$ in the statement. One could recover the radius $r/8$ by adding an additional rescaling and covering argument, or alternatively one could keep the smaller radius and subsequently adjust all the ones in the rest of the statements.

(4) In the derivation of the second inequality at line 7 of page 12 we are implicitly assuming $\theta_i(0) \geq 1$. This is an instance where the assumption $x_0 \in \text{spt}(\mu_V)$ in Theorem 3.2 is used: the inequality on the density follows from Proposition 2.2 because we should assume $0 \in \text{spt}(\mu_{V_i})$.

(5) The argument for the bound on $\mathcal{H}^k(F)$ in formula (5.4), specifically the first inequality, is incomplete. A complete argument is achieved remarking first that, because $\theta = 1$ \mathcal{H}^k -a.e. on $\Gamma \cap B_{1/8}$, $\mathcal{H}^k(F) = \mu(F)$. We can then use the inequality

$$\mu(F) \leq \sum_i \mu(B_{\rho_i}(x_i)).$$

Under the assumption that $5\rho_i \leq \frac{1}{2}$ we can then get to desired upper bound because it was already shown that

$$\mu(B_{\rho_i}(x_i)) \leq (\omega_k + \lambda)r^k.$$

In the way G is defined it is however quite possible that $5\rho_i \geq \frac{1}{2}$. Its definition should therefore be adjusted to

$$G := \left\{ x \in \Gamma \cap B_{1/64} : E(V, \pi, x, \rho) \leq \lambda \quad \forall \rho \in (0, \frac{1}{16}) \right\}.$$

The result is that we should adjust the radius in the statement of Proposition (5.1) as indicated in point (3) above.

(6) Page 17, line 2 from below: $\partial_{y_j} f_m$ should be $\partial_{y_m} f_j$.

(7) A space is missing between “missing” and (6.16).

(8) Page 20, four lines from below, the two 2^{-k} factors should in fact be 2^k .

(9) Page 21, line 3, the expression $x \in B_{1/4}$ should be replaced with $x \in B_{1/4} \cap \Gamma$ (which then guarantees $\mu_V(B_r(x)) \geq C^{-1}r^k$ in the following lines).