A universe polymorphic type system. Inductive types.

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To have one syntactic category "term" instead of two as in the original suggestion let us consider the system where there is an additional constant $Type$. Now every term will have a type with what was previously called "type expressions" becoming the terms T such that $\tau_\Gamma(T) = Type$. We set $\tau_\Gamma(Type) = Type$.

By construction, for any o one has $\tau_\Gamma(\tau_\Gamma(o)) = Type$.

The substitutional equivalence relations on terms will be such that:

1. The subset of terms that do not contain $Type$ is closed under these relations. Such terms will be called small terms,
2. The subset of terms of the form $(\prod(T_n, x_n. ..(\prod(T_1, x_1.Type))..))$, when $n = 0$ this expression is to mean simply $Type$, where $T_n, \ldots, T_1$ are small is closed under these relations. Such terms will be called P-terms.

Consider the inductive type declaration: `Inductive TI ( a1 : A1 ) .. ( an : An ) : forall ( a1' : A1' ) .. ( an' : An' ) , S :=`

There will be the following two cases:

1. (c1) $S = \mathcal{U}_u$ where u is a u-level expression.
2. (c2) $S = Type$.