Notes to the paper "Combinatorial structure of type dependency" by Richard Garner.

by Vladimir Voevodsky, started Jan. 18, 2015

On Definition 2.

Let F_W be the functor $X \mapsto \amalg_{w\in W} \amalg_{n\in \mathbf{N}} X^n$. Then for a set V, the set W^* defined in Definition 2 (relative to the set of names of variables V) is the value of the free monad M generated by the functor $X \mapsto (F_W X) \amalg V$ on the , or equivalently, the value of the free monad generated by F_W on V.

A missing Lemma 2'

Let Λ be a collection of judgements over an alphabet W and $D \subset \{w, p, s\}$ be a decent subset. Then there exists the smallest collection of judgements $\Lambda^{*,D}$ over the same alphabet such that:

1. $\Lambda^{*,D}$ is deductively closed relative to D,
2. for any judgement $J \in \Lambda$, one has $J \in \Lambda^{*,D}$ if for all $J' \in \partial(J)$ one has $J' \in \Lambda^{*,D}$.

Proof: There exists a collection of judgements that satisfies the conditions 1,2 of the lemma. Indeed, the collection of all judgements over W satisfies these two conditions.

The intersection of a family of subsets each of which satisfy the condition 1 again satisfies the condition 1 - the intersection of a family of subsets deductively closed relative to any D is deductively closed relative to the same D.

The intersection of a family of subsets each of which satisfy the condition 1 again satisfies the condition 1 the implication is easy to check.

Missing Definition 2"

Let Λ be a collection of judgements over an alphabet. The smallest collection $\Lambda^{*,D}$ defined by Lemma 2' is called the collection of derived judgements of Λ relative to D.

Missing example:

1. Suppose that Λ does not contain elements of the form $\vdash T\, type$. Then $\Lambda^{*,D}$ is empty for any D.

Indeed, judgements of this form are the only ones with the empty $\partial(J)$ and if all $\partial(J)$ for $J \in \Lambda$ are non-empty then the empty collection satisfies the conditions 1,2 of lemma 2'.

Missing Lemma 2"'

Let W, D and Λ be as above and suppose that $\Lambda \subset \Lambda^{*,D}$ then $\Lambda^{*,D}$ is the smallest collection that is deductively closed relative to D and contains $\Lambda \cup \partial(\Lambda)$.

Proof: If $\Lambda \subset \Lambda^{*,D}$ then $\Lambda^{*,D}$ is also the smallest class that satisfies the conditions of 1,2 of Lemma 2' together with the condition that the class contains Λ. This triple of conditions is equivalent to the triple of conditions that the class is D-closed, contains Λ and contains $\partial(J)$ for each $J \in \Lambda$.