A note on the $(\infty, 1)$-categories in the UF

Started January, 2017

1 Thoughts on $(\infty, 1)$-categories, January 21, 2017

1. the type M of $(\infty, 1)$-categories,
2. a function $Ob : M \to UU$ that we use as a coercion,
3. for any $X : M$, $x1, x2 : X$, $Mor(x1, x2) : UU$, units $1_x : Mor(x, x)$, compositions $Mor(x1, x2) \to Mor(x2, x3) \to Mor(x1, x3)$,
4. a function $Gr : UU \to M$ of h-level 1,
5. $Ob(Gr(T)) = T$,
6. the function $idtomor : paths\,T\,t1\,t2\ \to Mor_{Gr(T)}(t1, t2)$ defined by 1_t is an equivalence,
7. $M0 : M$ – the $(\infty, 1)$-category of types,
8. $Ob(M0) = UU$,
9. for $T1, T2 : UU$, $Mor_{M0}(T1, T2) = (T1 \to T2)$, 1-units are the identity functions and 1-composition is the usual composition,

Do we have a solution to the problem of constructing such a collection of data?

The first attempt, to take $M = UU$ does not work because of the very last condition.

The second attempt is to take M to be the type of precategory_data. That is an element of M is a type $Ob(C)$, the types of morphisms, the identities and the compositions. The function Gr is easy to define. The element $M0$ is defined as given in the conditions. This seems to satisfy all requirements. Therefore, more requirements are needed.

Let us add the $(\infty, 1)$-category of $(\infty, 1)$-categories.

1. $M1 : M$ – the $(\infty, 1)$-category of $(\infty, 1)$-categories,
2. $Ob(M1) = M$,
3. for $C1, C2 : M$, $functor\_coherence : functor\_data\,C1\,C2 \to UU$,
4. for $C1, C2 : M$, $Mor_{M1}(C1, C2) = total2\,functor\_coherence$,
5. for $C : M$, $id\_functor\_coherence : functor\_coherence\,id\_functor\_data$,
6. for $C1, C2, C3$, $functor\_comp\_coherence : forall(F1 : functor\,C1\,C2)(F2 : functor\,C2\,C3), functor$
7. for $C1, C2, C3$, $F1 : functor\,C1\,C2$, $F2 : functor\,C2\,C3$, $functor\_comp\,F1\,F2 = totla2\,(functor\_comp\_coherence\,F1\,F2)$.

This is still possible to solve with $M = precategory\_data$. Let us add representable functors.

1. for any $C : M$, $X : Ob(C)$ an element $Yo(X) : Mor_{M1}(C, M0)$,
2. for any C the function $Yo : Ob(C) \to Mor_{M1}(C, M0)$ is of h-level 1.

The last condition probably does not hold for $M = precategory\_data$.

1. $cat1 : M$ – the $(\infty, 1)$-category of 1-categories,
2. $Supp(cat1)_0 = Precategories$,
3. $Supp(cat1)_1(X, Y) = FunctorsXY$,
4. $Supp(cat1)_2(F, G) = FunctorMorphismsFG$,
5. $dim(Supp(cat1)) = 2$,
6. $M1 : M$ – the $(\infty, 1)$-category of $(\infty, 1)$-categories,
7. $Supp(M1)_0 = M$,