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A topos perspective on state-vector reduction. (English summary)

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Over the past 7 or 8 years, Isham has been developing interesting topos-based approaches to the subtle problems of quantum theory in general, and quantum gravity in particular. This paper begins as a simplification of an earlier paper by the same author. Truth values are assigned to expressions in quantum theory by means of sieves on a certain category. This is replaced, in the present article, by the simpler notion of monoids and left ideals. Given a monoid, M, which is going to be viewed as acting and some objects X, we can construct the category BM consisting of left M-sets. With this, "truth values" (a Heyting algebra) can be defined. The "truth object" in the topos is $\Omega = LM$, where LM denotes the set of left ideals of M. He then proceeds to show how classical propositions can be captured using the monoid $M = C^{\infty}(\mathbb{R}, \mathbb{R})$, while his earlier results in quantum theory follow from $M = \mathcal{M}(\mathbb{R}, \mathbb{R})$—the monoid of bounded measurable functions. He finally continues to provide a new view on state reduction. The monoid here is the monoid of all finite products of self-adjoint operators or strings of projectors. This has, by the way, an interesting relationship with Galois connections, which he mentions in a couple of footnotes. As always, Isham's papers are well written and very clear and thought provoking.

Reviewed by *Frank Antonsen*