Tfc-terms for CIC

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Abstract: A key function of current projects in formalization of mathematics should be the generation of ideas for the improvement of the available proof assistant systems. In this short paper I would like to suggest an addition to the Calculus of Inductive Constructions and more specifically to the type system of the proof assistant Coq inspired by my work on formalization of mathematics in the context of the univalent semantics. The proposed addition consists of a new term constructor and an associated with it computation rule. The terms produced by this constructor are called tfc-terms where tfc stands for "trivial fibration/cofibration".

Introduction   In the work on formalization mathematics I sometimes encounter situations where I am forced to make a choice with both branches being somewhat less than satisfactory. I call different kinds of such situations "tension points". There are various tension point in the constructive formalization of mathematics using Coq. Many of those are related to the interaction between constructivity and extensionality. The one I am addressing in this notes has the advantage of having a simple resolution.

I have first encountered the problem in question in the form of computationally inconvenient behavior of functions on set-quotients of types. Set quotients of types are defined in the univalent formalization as follows.

and while we will trace it back to an issue which has nothing to do with set-quotients I will start by describing what I mean by "computationally inconvenient behavior".

The description of the tfc-terms and of the associated computation rule