

# Some constructions which are difficult to make work in Coq

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Started November 29, 2012. Work in progress.

## 1 Eliminator for natural numbers from eliminator for W-types

We will use the notations of Coq. The classic W-types are defined as follows:

Inductive  $W (B : Type) (D : B \rightarrow Type) := wconstr : \text{forall } b : B, \text{forall } (f : D b \rightarrow W B D), W B D .$

It's eliminator (recursor) as produced by Coq looks as follows:

$W\_rect : \text{forall } (B : Type) (D : B \rightarrow Type) (P : W B D \rightarrow Type), (\text{forall } (b : B) (f : D b \rightarrow W B D), (\text{forall } d : D b, P (f d)) \rightarrow P (wconstr B D b f)) \rightarrow \text{forall } w : W B D, P w$

Let us try now to construct natural numbers as a particular case of W-types. We should take

$$B = unit \amalg unit$$

$$D(ii1(tt)) = \emptyset$$

$$D(ii2(tt)) = unit$$

Lets try to do it in Coq: