

Notes on type systems

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Started December 27, 2009

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The principal line of development in the theory of type systems and in mechanized reasoning in general is to enable faster and more efficient workflow without compromising the level of certainty in the proofs. Two main levels of certainty in the consistency of a type system are achieved by two different approaches - syntactic proofs of normalization theorems and constructions of models with values in other theories with high certainty level of consistency. On the other hand the power of a given system as a mechanism for proof development is determined, on the fundamental level, by the set of basic constructions and axioms available in the system.

The goal of the first phase of this project is to describe the "equivalence axiom" for the Martin-Lof type theory and to construct a "univalent model" of this theory which takes values in the homotopy category and satisfies this axiom. This both strengthens the Martin-Lof type theory in a highly non-trivial way and suggests a new way to formalize mathematics in this system.

The next stage is to incorporate the equivalence axiom in a Martin-Lof type system in a constructive way creating a new more expressive but still strongly normalizing system. This would open up a way to use categorical and then higher categorical reasoning in constructive mathematics without compromising its constructive nature.