# Dynamic logic. Stage 1

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## 1 The seed

1. Basic p-classes:
   (a) $E$ - class of elements
   (b) $S$ - class of small sets
2. Basic p-constructions:
   (a) $c : E \times E \to E$ - concatenation
   (b) $\nu : S \to E$ - naming
   (c) $\eta : S \to \bar{S}E$
3. Basic properties:
   (a) $c$ is a monomorphism
   (b) $c$ is associative
   (c) $\nu$ is a monomorphism
   (d) $\eta$ is a defining monomorphism
   (e) $\emptyset \in S$
   (f) Let $s : E \to \bar{S}E$ be the singleton map. Then $s(E) \subset S$.
   (g) Let $u : \bar{S}E \times \bar{S}E \to \bar{S}E$ be the union map. Then $u(S \times S) \subset S$.
   (h) Let $A \in S$ and $B \subset A$, then $A \in S$.
   (i) Let $A \in S$ and $2^A \in \bar{S}S$ the set of subsets of $A$. Then $\bar{S}\nu(2^A) \in S$.

## 2 Beginning of the story

**Definition 2.1** *[***md1***]*

1. *A set is an object of* $\bar{S}E$
2. *A small set is an object of* $S$

**Definition 2.2** *[***d2***] Product construction is a morphism* $prod : barSE \times \bar{S}E \to \bar{S}E$ *defined as the composition*

$$\bar{S}E \times \bar{S}E \to \bar{S}(E \times E) \stackrel{\bar{S}c}{\to} \bar{S}E$$

*It is denoted by* $(A, B) \mapsto A \times B$*.*

**Lemma 2.3** *[**l1**] The product construction is strictly associative i.e.*

$$prod \circ (prod \times Id_E) = prod \circ (Id_E \times prod).$$

**Proof**: ???

**Definition 2.4** *[**md3**] An embedded pair of sets $(A \subset B)$ is an object of $\bar{S}E \times \bar{S}E$ which belongs to inclusion subobject $Incl \subset \bar{S}E\bar{S}E$.*

**Definition 2.5** *[**md3**] A correspondence (between sets) is an object $(A, B, \phi)$ of $\bar{S}E \times \bar{S}E \times \bar{S}E$ which belongs to*

$$Cor\bar{S}E = (prod \times Id_{\bar{S}E})^{-1}(Incl)$$

*i.e. such that $\phi$ is a subset in $A \times B$.*

**Definition 2.6** *[**md4**] A map from a set $A$ to a set $B$ is an object of $Maps\bar{S}E \subset Cor\bar{S}E$ where $(A, B, \phi) \in Maps$ iff $\forall a \in A \exists ! b \in B$ such that $c(a, b) \in \phi$.*