Comments on:

Voevodsky C-system of a module over a Jf-relative monad 2016_01_31_Csystemfromamonad.pdf

The introduction is much improved since the previous version, and the editor may wish to read it. The current paper has 32 pages, and is part of a longer sequence of papers designed to prove the soundness of univalent type theory in the presence of the univalence axiom, the law of the excluded middle, and the axiom of choice. Part of the proof concerns the realization of the univalence axiom as a property about simplicial sets, and part of the proof relates the formal language of univalent type theory to a mathematical structure of its syntax, which can them be modeled in topology to prove soundness. This paper covers some of the latter part.

The long study of the mathematics of computer language syntax has resulted in 5 equivalent notions, all of which are treated in detail or mentioned in this paper:

- l-bijective C-systems - Lawvere theories - Jf-relative monads - abstract clones - substitution systems (or substitution algebras?)

Type theoretically they all correspond to type theories where there is just one type, which might be taken to be the type of raw terms underlying the (more interesting) type theory to be studied. One expects the formalism of the raw syntax of terms to be elementary, so the proliferation of these concepts is unexpected and confusing.

The isomorphism between the category of l-bijective C-systems and the category of Lawvere theories is described in Voevodsky's 15 page preprint [17]. The equivalence between the category of Lawvere theories and the category of Jf-relative monads is established in Voevodsky's 21 page preprint [21]. The current paper uses both of those to perform some computations.

The mathematics in the current paper is straightforward, the proofs are easy, and the concepts are concrete, yet bulky and unfamiliar to mathematicians, since they are oriented toward dealing with the syntax of formal languages in a mathematical way. Thus the typical mathematician will find the paper rough going, since it is long, and the main reward from reading it is to come far in the future, after the "initiality conjecture" is proved, thereby establishing the soundness of univalent type theory.

Theorems 5.12 and 6.10 are the only theorems of the paper, and each one is simply a long list of formulas. I didn't read them carefully or check them – the proofs are presented in detail. We take on faith that the formulas will be useful later, and are offered this reassurance in the introduction: "In the next paper we will connect these computations to the conditions that the valid judgements of a type theory must satisfy in order for the term C-system of this type theory to be defined". I ask for more details in my remarks below.

I expect the initiality conjecture will be proved eventually, and when that happens, this paper will be an important part of the proof of the soundness of univalent type theory, which will be an essential and important foundational result in the field.

The main question in my mind, as referee, is whether the preprints (there are nine at this point), including this one, should instead be combined into a large future monograph that would contain the crowning theorem. I've mulled this over for some minutes and decided "no" – the individual preprints, while not offering much reward for reading them now, offer accessible segments of the whole, each with a certain unity to them. There are also some advantages to having the work reach the devoted followers early, as its pieces are accomplished. Think of them as chapters in a serial novel.

I recommend publication in JPAA.

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MATHEMATICAL REMARKS:

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the introduction: "In the next paper we will connect these computations to the conditions that the valid judgements of a type theory must satisfy in order for the term C-system of this type theory to be defined". Could something more be said here to give the reader an inkling of why such formulas might be useful? For example, is the form of the formulas somehow closer to the way inference rules for a type theory are traditionally formulated?

What is the type-theoretic motivation for the construction CC[F], introduced on page 7? Oh, the answer comes much later, on page 25: "The role of these C-systems in the theory of type theories is that the term C-systems of the raw syntax of dependent type theories are of this form and therefore the term C-systems of dependent type theories are regular sub-quotients of such C-systems and can be studied using the description of the regular sub-quotients given in [20]." It would be good for the reader to have an indication of that earlier, perhaps all of it, or perhaps something along the lines of "CC will capture the raw syntax of terms of a type theory, F will capture the raw syntax of types, and CC[F] will encode them together", if I have that right.

Jf is well behaved in the sense of [2, section 4], so any relative monad for it extends to a monad on the category of sets, as is proved there. This seems to answer the question in remark 4.6, which is reserved for a future paper, unless the proof there implicitly uses LEM.

Near "Example 6.2 An important example of LM is given by ..." it might be good to say why it's important, from the point of view of type theory. Does it have to do with the possibility of not distinguishing initially between terms for elements and terms for types, throwing them all into CC, and then using the construction CC[LM] to create the C-system for the raw syntax of contexts?

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MINOR STYLISTIC REMARKS:

the o in Martin-Lof in the bibliography is typeset incorrectly

in "For the two of the several main groups of inference rules" omit the first "the"

Change "This allows to avoid the use of natural numbers in some of the arguments that significantly simplifies the proofs" to "This allows us to avoid the use of natural numbers in some of the arguments, thereby significantly simplifying the proofs."

Change "The C-systems of this form remind in some way the affine spaces over schemes in algebraic geometry" to "The C-systems of this form remind us in some way of affine spaces over schemes in algebraic geometry."

remove hyphens from "sub-space" and "sub-quotient"

remove "very" from "not very interesting"

add the indicated comma in "While the geometry of affine spaces in itself is not very interesting, their sub-spaces encompass all affine algebraic varieties of finite type."

remove spaces before periods at ends of sentences: "... finite type ."

change "look to be" to "seem to be"

change "not very different from CC" to "similar to CC". The reason is that "not very different" is ambiguous: it could be the negation of "very different", or it could mean "only slightly different". Also, "very" is an overused qualifier in English prose, often used with no clear intent.

I find "Regular sub-quotients of any C-system CC are classified by ... " abrupt because "regular subquotients" are not mentioned in [20]. Of course, one can guess what it is, but why make the reader guess? At this point, it is probably useful to remind the reader that the result stated is covered in [20], if it is, and to provide a citation. If it's not in [20], then why is it true?

change "we first remind the notion of a relative monad on" to "we first recall the notion of a relative monad on". Every use of "remind" without mentioned the persons being reminded should be replaced by "recall", or the persons should be mentioned: "We recall the notion X"; "We remind the reader of the notion X".

add the indicated commas: "Since this paper, as well as other papers in the series on C-systems, is expected to play a role in the mathematically rigorous construction of the simplicial univalent representation of the UniMath language and the Calculus of Inductive Constructions, and since such a construction itself can not rely on the univalent foundations, the paper is written from the perspective of the Zermelo-Fraenkel formalism."

in "The paper is written in the formalization-ready style" the second "the" is inappropriate, because there can be multiple styles ready for formalization. Avoid hyphens. How about this?: "The paper is written in a style that should lend itself to straightforward formalization."

The sentence "In all that follows we write Sets instead of Sets(U)" would be clearer if one didn't have to guess the meaning of "Sets(U)", and if "Sets" hadn't already been written previously. Just say that "We let Sets denote the category of sets in U", or give a forward reference to the definition of Sets(U) in the body of the paper.

change "Recall that for a C-system CC, and object of CC such that l() ¿ i we ..." by inserting "an", inserting a comma, and removing a comma, to get "Recall that for a C-system CC and an object of CC such that l() ¿ i, we ...". The reason for removing the first comma is that the "and" is separate two noun phrases, not clauses. The second comma helps the tired reader.

change "... define a pair of an object and a morphism defined inductively as ..." to "... denote the object and the morphism defined inductively by ...". The point is that a definition associates notation to a mathematical object, after which the notation denotes the object. Moreover, neither notation refers to a pair, we have just two notations for two objects.

In "If is over " it seems that "is over" is being used instead of " ¿ " or "≥", but the reader must guess it, as it is nonstandard, so define it. Better than "over" would be "above", because "over" already has a meaning in any category, e.g., a scheme X over S.

In the statement of lemma 2.2, the reader can deduce that Gamma' and Gamma" are over Gamma"', because "a" is, but why make the reader do that? Better would be to start with a over Gamma"' and Gamma"' over Delta, and deduce that a is also over Delta.

It seems odd that "Construction 3.2" is the construction corresponding to "Problem 3.1". Why not use the same number for both? Then it would be clear to the reader that they correspond.

change "where () is the unique element of the one point set that is the product of the empty sequence" to "where () is the unique element of the one point set, the empty sequence", because "product of the" is not applicable here.

before "which is well defined because" put a comma.

in remark 3.5 C[F] should be CC[F]

in remark 3.6 mathematicians will have to guess that "unit" refers to the one element set

change "remind" to "recall" in "Let us remind it here."

This word order is clumsy: "The set of Jf-relative monads is in an easy to construct bijection with the set of abstract clones as defined in [6, Section 3]." I suggest "There is a bijection between the set of Jf-relative monads and the set of abstract clones as defined in [6, Section 3]." Avoid saying "easy" or "straightforward" whenever possible, since readers differ in ability.

It might be good to say that Construction 4.3 occurs in [2].

In problem 4.4 a reference is given to p.133 of [10]. The reader must guess that the reference is given as a source for the definition of "monad", and will be misled by the use of "cf." when there is nothing in the text to compare with what's in the reference. See "https://en.wikipedia.org/wiki/Cf.". Perhaps simply precede the problem with a sentence saying that the definition of monad can be found in [10, p. 133].

The paragraph beginning "Recall that the category T has as ..." will seem overly pedantic to a mathematician, who will be comfortable with the looser phrasing "we identify ((m,n),f) with f when it will cause no confusion", and will take it to imply the needed coercions.

In "The proof of the composition and the identity axioms of a functor are easy" we have the subject "proof", which is singular, and the verb "are", which is plural. Avoid saying "easy" or "straightforward" whenever possible, since readers differ in ability. Say just that the proof is omitted or that it is left to the reader.

change "We will only consider the case when U is a universe" to "We will consider only the case when U is a universe."

Remark 4.8 can be safely eliminated.

Change "the opposite category to T" to "the opposite category of T".

Insert the indicated comma: "previous section, the most important of which ..."

This sentence is impossible to understand: "In the final Section 6 we apply the construction of Section 3 to C(RR) taking into account that the functors LM : C(RR)op Sets are the same as the functors K(RR) Sets that are the same as the relative (left) modules over Jf."

Clearer than "In (25) and Construction 6.8 we compute the B-sets B(C(RR,LM)) and B (C(RR,LM)) and in Theorem 6.10 the action of the B-system operations on these sets." would be "In (25) and Construction 6.8 we compute the B-sets B(C(RR,LM)) and B (C(RR,LM)), and in Theorem 6.10 we compute the action of the B-system operations on these sets."

The sentence "Combining this conjecture with Theorem 5.12 we conclude ..." sounds strange, because conjectures don't allow you to conclude anything. Maybe say something like "The assumption that the conjecture is true, along with Theorem 5.12, would allow us to conclude ..."

The author writes "the category of abstract clones is equivalent to the category of substitution systems of [6, Definition 3.1]." But the cited definition defines "substitution algebras", not "substitution systems".

Remove "the" from "left modules over ... are the presheaves on C(RR)" and from "i.e., the contravariant functors ... ", or insert it before "left modules".

Remove "the" from "can be thought of as the substitution."

Near "Modules (actually left modules) over relative monads were introduced in [1, Definition 9]" it might be good to attribute the notion of left modules over monads to the appropriate paper of Hirschowitz and Maggesi, as was done in the first version of the paper.

The sentence "One can verify that for any Jf-relative monad RR, = where is the permutation of 0 and 1 in stn(2)" doesn't quite make sense, because : RR(2) -¿ RR(2), whereas : stn(2) -¿ stn(2). One could say instead that "One can verify that for any Jf-relative monad RR, is induced by the transposition of 0 and 1 in stn(2)."

In "we have for each m,n N a function R(n,m) ..." replace "R(n,m)" by "RR(n,m)".

In the remark "we can define a left l-module lLM over RR as a quadruple ..." I wonder what "l" refers to. I realize that there is also something called "l" in the following definition, but that can't be the same "l", or else it would been introduced earlier. I'm also confused because one would expect terms such as "left module over RR" and "left RR-module" to be used instead, and to be equivalent.

We also see "These l-versions of the relative monads and their modules ... " and are in the dark about what an "l-version" of something is.

That remark continues: "we can define a left l-module lLM over RR as a quadruple ... where operations l, and LM satisfy some conditions. Once these conditions are properly established ... ". The conditions are never listed. It is important to warn the reader at an earlier stage by saying something like this instead: "we would like to be able to define a left l-module lLM over RR as a quadruple ..."

change "for a particular class of the inference rules" to "for a particular class of inference rules"

This sentence tantalizes: "The term models for a class of type theories can be obtained by considering slices of the term model of the type theory called Logical Framework (LF), but unfortunately it

is unclear how to extend this approach to type theories that have more substitutional (definitional) equalities than LF itself." Is the work published? If so, where? Who did the work?

in "obtained by considering slices of the term model of the type theory called Logical Framework" is the word "slices" being used in a technical sense? If not, it's confusing. If so, a footnote giving a reference to the definition would be appropriate, or one could rephrase the sentence to make it self-evident how to read it.

Change "At the time when that paper was written" to "At the time that paper was written" or to "When that paper was written", to eliminate redundancy.

insert the indicated comma: " body of work on type theory, which is reflected "

in "(cf. also [9, Example 1.2.3] that claims as ... " I suspect "cf." is being used to mean "see". If so, change it to "see", because "cf." means "compare". Then put a comma before "that" and change "that" to "which".

I see both "relative monad *on* a functor" and "relative monad *over* the functor". It would be good to pick "on" or "over" and to use it uniformly.

In "In the next paper we will connect these computations to the conditions that the valid judgements of a type theory must satisfy in order for the term C-system of this type theory to be defined" change "this" to "the", or better, rephrase as "In the next paper we will connect these computations to the conditions that the valid judgements of a type theory must satisfy in order for its term C-system to be defined."