

Dear Volodya, I understand that you are quite engaged into mathematics and don't have neither time nor wish to read through the final version of the text. Here is however something I would really want you to do. I include below the introduction to the paper. Please read it through carefully and make any changes you find necessary. I also include the referees report. I presume that I took into account all his remarks (not that there were many of them) except for the transitivity of the Gysin morphism. I am more or less certain that this is the case and that we do not really need it. I suggest that we include this as a remark without proof and leave it at this.

All the very best Andrei

Referee report on the paper

"Bloch-Kato conjecture and motivic cohomology with finite coefficients" by A. Suslin and V. Voevodsky.

In my opinion, the arguments of the paper are correct and complete. I have a few minor suggestions and comments, which are :

Gysin morphism: Do the authors ever prove (or need) functoriality of the Gysin morphisms for a string of inclusions $Z' \subset Z \subset X$? Perhaps this is not necessary, but it is very easy to assume this without thinking in the arguments.

Resolution of singularities: It would be nice if the authors would pin down exactly why the quasi-invertibility of the Tate object is needed. It seems that this is used to lower weights when passing to cohomology with supports, but it would be nice to know if this is the only place.

pg. 43, l.-5: ... that the complex of Nisnevich sheaves... (instead of "a" complex)

In this paper we show that the Beilinson-Lichtenbaum Conjecture which describes motivic cohomology of (smooth) varieties with finite coefficients is equivalent to the Bloch-Kato Conjecture, relating Milnor K -theory to Galois cohomology. The latter conjecture is known to be true in weight 2 for all primes [M-S] and in all weights for the prime 2 [V 3].

The paper is organized as follows. In the first five sections we remind the definition and main properties of motivic cohomology, following the approach developed in [V 1, V 2, F-V]. This part of the paper may be viewed as a general introduction to the motivic cohomology theory. The proof of the main Theorem occupies sections 6-10. In section 11 we show that the Beilinson-Lichtenbaum Conjecture may be further reduced to vanishing of certain Bockstein operations in étale cohomology. In the special case $p = 2$ this fact was previously proved by Merkurjev [Me]. Finally the last section is devoted to the proof of a somewhat technical, but quite important result concerning the finiteness of the cdh-cohomological dimension of Noetherian schemes. This fact is crucial in dealing with motivic cohomology of non smooth varieties and we would like to express our gratitude to O. Gabber who helped us to work out the proof.

It should be noted that the proofs of all the most important results contained in this paper require resolution of singularities to hold over the base field, so that strictly speaking our Main Theorem so far applies only in the characteristic zero case. It's not hard to see however that the only place where the use of the resolution of singularities is absolutely inevitable (if one follows our approach) is the

proof of the purity theorem, which allows to replace motivic cohomology of weight n with supports in a smooth subscheme Z of codimension d by motivic cohomology of Z of weight $n - d$. The last property holds without the assumption of resolution of singularities if one defines motivic cohomology in terms of higher Chow groups [B, B1]. Probably the best course would have been to define motivic cohomology of weight n using the sheaf of equidimensional over X cycles in $X \times \mathbb{A}^n$ of relative dimension zero. The resulting complex $\mathbb{Z}'(n)$ is quasiisomorphic to $\mathbb{Z}(n)$ in case resolution of singularities holds [F-V]. Moreover the corresponding motivic cohomology groups always coincide with the higher Chow groups - cf. [Su]. Thus defining motivic cohomology as Zariski hypercohomology with coefficients in $\mathbb{Z}'(n)$ we will have the nice functorial properties and flexibility of motivic cohomology, while at the same time we will have also localization and purity Theorems valid without any assumptions on the base field. We did not try to pursue this course, since on the one hand we wanted our exposition to be more or less self contained and on the other hand a similar approach was already used in a recent preprint of M. Levine and Th. Geiser, who work directly with higher Chow groups and prove (using essentially the same ideas) a Theorem similar to our Main Theorem, but valid over arbitrary fields.

The first version of this paper appeared in 1995 and contained several minor errors and inaccuracies . We are much obliged to all who pointed out these inaccuracies and errors to us. Special thanks to B. Kahn and Ch. Weibel.

The first author would like to express his gratitude to IHES for the hospitality he was enjoying while working over the final version of this paper.