

Report about

Subsystems and regular quotients of C-systems

by **V. Voevodsky**

The paper is about some properties of the notion of “contextual category” introduced by J. Cartmell in the seventies. The author renames that notion as ”C-system” because it is not preserved by equivalences of categories and it is more of an algebraic nature. Then he presents the notion of C-subsystem and that of quotient under a regular equivalence relation for any given C-system.

I recommend publication after correcting the references, clarifying the motivations for introducing C-subsystems and suitable quotients of C-systems and presenting the notion of homomorphism between C-systems (which is mentioned in the introduction but it is not given in the paper).

It would be also interesting to point out the ways in which one can get a C-system from a category with an arbitrary choice of pullbacks like for instance that in

Hofmann M. On the interpretation of type theory in locally Cartesian closed categories, Computer Science Logic, Lecture Notes in Computer Science Vol. 933, p. 427-441, 1995

The critical issue is how to determine a C-system out of a finitely complete category with an arbitrary choice of the structure but in an essentially algebraic way.

Another issue is how to determine a C-system out of a finitely complete category with no choice of pullbacks as in an exact completion introduced in

Carboni, A. and R. C. Magno. The free exact category on a left exact one. Journ. Austr. Math. Soc. vol. 33, p. 295-301, 1982

with relevant examples in

Carboni, A. Some free constructions in realizability and proof theory. Jour. Pure Appl. Alg., Vol. 103, Issue 2, p.117-148, 1995

or in a quotient completion as in

Maietti M.E. and Rosolini G. Quotient completion for the foundation of constructive mathematics. Logica Universalis, Vol.7, Issue 3, p.371-402, 2013

Minor comments and typos:

1. Why not using the name ”Contextual-system” in place of ”C-system”?
2. p. 1 last but one paragraph a parenthesis is missing
3. p. 1 4th paragraph: what is $(Ob(CC), \tilde{Ob}CC)$? You should say what these sets mean.
4. it should be convenient to use the standard notation of composition (opposite to that in the paper)

5. p.1 l.2 in section 1: the reference [3, Def1.2, p.47] is not present in the bibliography
6. in section 2 it could be useful to give the notion of morphism between C -systems and C_0 -systems
7. p. 3 in def. 2.2 it would be useful to draw a diagram
8. p. 4 remark 2.4 and also remark 2.6 are not clear if you do not give the notion of homomorphism between C -systems
9. p.4 l.16 does [3] refer to Streicher?
10. in section 3. it could be useful to observe that s_f is in \tilde{Ob} for each f
11. last line of 5 typo: write \tilde{Ob} instead of \tilde{ob}
12. p.6 l.3 $p_X^*(X)$ is not a binary product $X \times X$ in CC but a fibred product...
13. in general the notation CC, CC' is confusing
14. the diagram in proposition 4.3 should be clearer and there should be $\tilde{T}(Y, r)$ and $\tilde{S}(s, r)$
15. in the first line p.7 is not since $s_f: Y \rightarrow X_f$, but since $ft^{l(X)}(f) \in Mor(C, C')$.
16. make proofs of lemmas 4.4, 4.5, 4.6, 4.7, 4.8 clearer (maybe with diagrams)
17. p. 7, l. 3 from below: $g^*(s_f)$ is not clear: the notation $f^*(g)$ is not defined in the paper
18. why do you call the congruence relation on a C -system "regular"? Explain the motivation.
19. p. 9 statement on lemma 5.3 has on "Id" without index.
20. p. 10 typo in the second paragraph: write the capital letter after a dot.
21. p. 10 third paragraph: make clear what predicate you prove by induction
22. p. 10 second display from below is not clear: write an inductive definition.
23. p.10 in the last display the occurrences of the equality symbol should be replaced by equivalences of propositions....