Course Syllabus for Math 287: Algebraic L-Theory and Surgery

Course Description Let X be a finite complex. When does X has the homotopy type of a smooth manifold? First, X must satisfy Poincare duality. Second, there should be a vector bundle T_X on X which plays the role of the tangent bundle. If the dimension of X is divisible by 4, then the signature of the intersection form on the cohomology $H^*(X)$ should be given as a certain characteristic class of T_X (the Hirzebruch signature formula). Amazingly, if X is simply connected and has dimension $4k > 4$, then these conditions are sufficient to guarantee that X is homotopy equivalent to a smooth manifold. In this course, we will study this theorem and some of its generalizations: to manifolds which are not assumed to be smooth, to manifolds which are not assumed to be simply connected, and to manifolds whose dimension is not assumed to be divisible by four. In order to obtain a classification theorem in these settings, we will develop the subject of *algebraic L-theory*, which can be regarded as an elaborate generalization of the classical Witt groups of quadratic forms.

Meeting Time MWF at 1.

Office Hours Thursdays 2-3, or by appointment.

Texts There is no textbook required for this class. Useful references include Ranicki's "Algebraic L-Theory and Topological Manifolds" and Wall's text "Surgery on Compact Manifolds."

Course Website http://www.math.harvard.edu/∼lurie/287.html

Prerequisites Familiarity with the machinery of modern algebraic topology (simplicial sets, spectra, ...). The first part of the course (where we develop the subject of L-theory) will require a high tolerance for abstraction, while the second part (where we apply the theory to classify manifolds) will be more concrete.

Topics likely to be covered

- Construction of L-theory spectra.
- Witt groups of quadratic forms.
- Techniques of surgery in algebraic and geometric settings.
- Poincare duality spaces and Spivak normal fibrations.
- L-theory orientations and generalized signature formulas.
- Classification of high-dimensional manifolds, up to h-cobordism.

Grading Undergraduates or graduate students wishing to take this course for a grade should speak with the instructor.