Math 155, Problem Set 3 (due September 26)

September 18, 2011

Let S and T be species. An *isomorphism* from S to T is a collection of bijections $f_I : S[I] \to T[I]$ such that, for every bijection of finite sets $\pi : I \to J$, we have $T[\pi] \circ f_I = f_J \circ S[\pi]$.

(1) Let S and T be species. Construct an isomorphism between the species $\exp(S+T)$ and $\exp(S)\exp(T)$.

(2) For each $n \geq 0$, let c_n denote the number of ways to partition the set $\{1, \ldots, n\}$ into subsets, each of which have odd size. Give a formula for the exponential generating function

$$\sum_{n \geq 0} \frac{c_n}{n!} x^n.$$

(3) For each $n \geq 0$, let p_n be the probability that a randomly chosen permutation of the set $\{1, 2, \ldots, n\}$ has order divisible by 3. Prove that $p_{n-1} \leq p_n$, and that the inequality is strict if and only if n is divisible by 3.