Math 155, Problem Set 10 (due November 21)

November 12, 2011

(1) Let $x_0, x_1, x_2, \ldots$ be a sequence of distinct real numbers. Show that the sequence $\{x_i\}$ has a monotone subsequence: that is, a subsequence which is either strictly increasing or strictly decreasing.

For every sequence of positive integers $m_1, \ldots, m_t$, let $R(m_1, \ldots, m_t)$ denote the corresponding Ramsey number: that is, the smallest integer n such that if G is a complete graph on n vertices, then every edge coloring of G using the set of colors $\{c_1, \ldots, c_t\}$ has monochromatic subgraph of size m_i and color c_i, for some $1 \leq i \leq t$.

(2) Let T be fixed. Show that

$$R(n, n, \ldots, n) \geq t^{\frac{n}{2}}$$

for all sufficiently large values of n.

(3) Let $m \geq 2$ be a fixed integer, and let $\delta > 0$ be a positive real number. Show that

$$R(m, n) \geq n^{\frac{m-1}{2} - \delta}$$

for all sufficiently large values of n.