Math 155, Problem Set 1 (due September 12)

September 5, 2011

(1) Find a partial fraction decomposition for the expression $\frac{1}{(1-x)(1-2x)(1-3x)}$: that is, determine constants α, β, and γ such that

$$\frac{\alpha}{1-x} + \frac{\beta}{1-2x} + \frac{\gamma}{1-3x} = \frac{1}{(1-x)(1-2x)(1-3x)}.$$

Use this to give a closed-form expression for the Stirling numbers $\left\{ {n \atop 3} \right\}$.

(2) For each integer $n \geq 0$, let c_n denote the number of ways to tile a 3-by-n board with 3-by-1 tiles (which may be placed vertically or horizontally). Give a formula for the generating function $f(x) = \sum_{n \geq 0} c_n x^n$.

(3) Let $\phi = \frac{1+\sqrt{5}}{2}$ denote the golden ratio. Prove that, for $n \geq 5$, the real number ϕ^n is within .1 of an integer. In other words, prove that there exists integers T_n with $|T_n - \phi^n| < .1$. (Hint: study the sequence of integers T_n, and try to find a closed-form expression for it.)