Math 114, Problem Set 5 (due Monday, October 21)

October 14, 2013

(1) Let $E \subseteq \mathbb{R}^n$ be a measurable set, and let $f_0 \leq f_1 \leq f_2 \leq \cdots$ be an increasing sequence of integrable functions on E for which the the sequence of integrals $\{\int_E f_i\}_{i \geq 0}$ is bounded. Show that the sequence $\{f_i\}$ converges almost everywhere to an integrable function f, and that $\int_E f$ is a limit of the sequence $\{\int_E f_i\}_{i \geq 0}$.

(2) Let $f : \mathbb{R} \to \mathbb{R}$ be an integrable function. Show that $\int_{\mathbb{R}} f$ is a limit of the sequence of real numbers $\{\int_{-n}^{n} f\}_{n \geq 0}$. Here $\int_{-n}^{n} f$ denotes the integral $\int_{[-n,n]} f|_{[-n,n]}$.

(3) Let $f : \mathbb{R}^n \to \mathbb{R}$ be an integrable function, and suppose that $\int_B f|_B = 0$ for every open box $B \subseteq \mathbb{R}^n$. Prove that f vanishes almost everywhere.

(4) Let E be the subset of $[0, 1]$ consisting of those real numbers whose decimal expansion contains infinitely many occurrences of the digit 7. Show that E is a measurable set, and compute its measure.