Math 114, Problem Set 3 (due Monday, September 30)

September 24, 2013

(1) Let $E \subseteq \mathbb{R}^n$ be a measurable set with $\mu(E) < \infty$. Show that for each $\epsilon > 0$, there exists a set $E' \subseteq \mathbb{R}^n$ which is a finite disjoint union of open boxes satisfying

$$\mu(E - E'), \mu(E' - E) < \epsilon.$$

(2) Let $f_1, f_2, \ldots : \mathbb{R}^n \to \mathbb{R}$ be a sequence of measurable functions and suppose that for each $\vec{x} \in \mathbb{R}^n$, the sequence $\{f_i(\vec{x})\}$ is bounded. Show that the function $f(\vec{x}) = \limsup\{f_i(\vec{x})\}$ is measurable.

Let $f : \mathbb{R} \to \mathbb{R}$ be a function. We say that f is *Borel measurable* if, for every real number t, the set $\{x \in \mathbb{R} : f(x) \leq t\}$ is Borel measurable.

(3) Prove that if $f, g : \mathbb{R} \to \mathbb{R}$ are Borel measurable functions, then the composition $g \circ f$ is Borel measurable.

(4) Let $f : \mathbb{R} \to \mathbb{R}$ be a measurable function. Show that there exists a Borel measurable function g which is equal to f almost everywhere.