

Math 114, Problem Set 10 (due Monday, November 25)

November 16, 2013

- (1) Let X and Y be metric spaces, let $f : X \rightarrow Y$ be a function, and let $\Gamma(f) = \{(x, y) \in X \times Y : f(x) = y\}$ be the graph of f . Show that if $\Gamma(f)$ is closed and Y is compact, then f is continuous.
- (2) Show that the subset $\mathbf{Q} \subseteq \mathbb{R}$ of rational numbers is not a G_δ -set (that is, it cannot be obtained as a countable intersection of open sets).
- (3) Let V be a Banach space. Show that the dimension of V is either finite or uncountable (that is, V does not have a countably infinite basis).
- (4) Let $E \subseteq \mathbb{R}^n$ be a measurable set with $0 < \mu(E) < \infty$. Let us regard $L^1(E)$ as a metric space, and $L^2(E)$ as a subset of $L^1(E)$. Show that $L^2(E)$ is meagre (that is, it is a countable union of nowhere dense subsets of $L^1(E)$).