# Exercise #3

<u>Goal</u> For some $\varepsilon = O\left(\sqrt{\frac{d}{|\mathbb{F}|}}\right)$

$$\mathrm{agr}_{\leq 2d}(f) \geq \left( \mathop{\mathbb{E}}_{S \in S_k^{k+1}} \left( \mathrm{agr}_{\leq d}(f_{|S}) \right) \right)^2 - \varepsilon$$

proved in class

$\Downarrow$

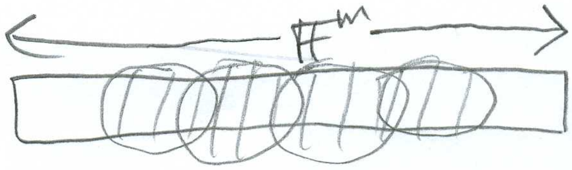
For some $\varepsilon = \left(\frac{d}{|\mathbb{F}|}\right)^{\Omega(1)}$,

$$\mathrm{agr}_{\leq d}(f) \geq \mathop{\mathbb{E}}_{S \in S_k^{k+1}} \left( \mathrm{agr}_{\leq d}(f_{|S}) \right) - \varepsilon$$

needed to conclude proof

$\uparrow$

4 Lemmas + conclude



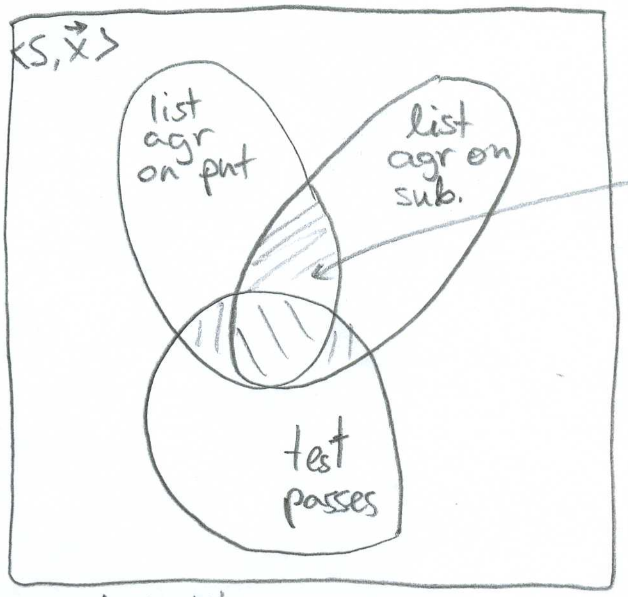

(1) LDT Thm

success of test $\Longrightarrow$ some agreement with low deg

You should prove that there exist <u>few</u> low degree poly that explain <u>almost all</u> the success of the test.

(2) different notion of "explaining": $\Pi(S) \equiv P_{1_S}$ (Note correction

agreement on the entire subspace, rather than on a point.



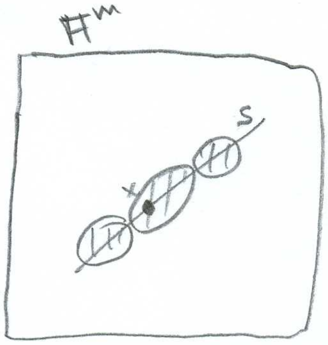

understand prob. of events & their intersections

probability space

(3) <u>Agreement increase</u>

II [ almost ~~everything~~ all agreement is explained by short list

III [ Is it possible that <u>within S</u> agreement with $\pi$ is small, yet the average over S agreement with $\pi$ is large?




I [ If only the weak LDT Thm is correct, then on average over S, agreement with $\pi$ is small.

## (4) Degree decrease

$q \in \mathbb{F}_{\leq 2d}[x_1, \dots, x_m]$

restrict $q$ to random $s \in S_k^{k+1}$.

What is the degree of $q_{|s}$? Can it be $< \deg q$ with non-negligible probability?