2-Block Springer Fibers and Knot Homology

Ben Webster
(joint with Catharina Stroppel)

IAS/MIT

December 13th, 2007

Outline

1 Geometry of Springer fibers
- Nilpotents in $\mathrm{Mat}(n, n)$
- 2-block nilpotents

2 Cohomology
- Components of Springer fibers
- Intersections of components

3 Convolution algebras
- Combinatorics
- Applications and generalizations

Nilpotents in $\mathrm{Mat}(n, n)$

Definition

We call an element $N \in \mathrm{Mat}(n, n)$ **nilpotent** *if* $N^n = 0$.

We denote the set of nilpotents by $\mathcal{N} \subset \mathrm{Mat}(n, n)$, and call it the **nilcone**.

Nilpotents in $\mathrm{Mat}(n,n)$

Definition

We call an element $N \in \mathrm{Mat}(n,n)$ **nilpotent** *if $N^n = 0$.*

We denote the set of nilpotents by $\mathcal{N} \subset \mathrm{Mat}(n,n)$, and call it the **nilcone**.

Definition

We say that a nilpotent N **preserves** *a flag $\underline{F} = (F_1 \subset \cdots \subset F_n = \mathbb{C}^n)$ if for each i, we have $N(F_i) \subset F_i$.*

Note that any nilpotent N preserves at least one flag in the flag variety $X = \mathrm{GL}(n)/B$.

Nilpotents in Mat(n, n)

Definition

We call an element $N \in \mathrm{Mat}(n, n)$ **nilpotent** *if $N^n = 0$.*

We denote the set of nilpotents by $\mathcal{N} \subset \mathrm{Mat}(n, n)$, and call it the **nilcone**.

Definition

We say that a nilpotent N **preserves** *a flag $\underline{F} = (F_1 \subset \cdots \subset F_n = \mathbb{C}^n)$ if for each i, we have $N(F_i) \subset F_i$.*

Note that any nilpotent N preserves at least one flag in the flag variety $X = \mathrm{GL}(n)/B$.

Definition

We define the Springer fiber of N to be the set of flags preserved by N.

$$X_N = \{\underline{F} \in X | N(\underline{F}) \subset \underline{F}\}$$

The Springer resolution

The nilcone $\mathcal{N}$ is very singular, but it has a resolution of singularities.

The Springer resolution

The nilcone $\mathcal{N}$ is very singular, but it has a resolution of singularities. Let

$$\tilde{\mathcal{N}} = \{(\underline{F}, N) | \underline{F} \in X, N \in \mathcal{N}, N(\underline{F}) \subset \underline{F}\}.$$

The Springer resolution

The nilcone $\mathcal{N}$ is very singular, but it has a resolution of singularities. Let

$$\tilde{\mathcal{N}} = \{(\underline{F}, N) | \underline{F} \in X, N \in \mathcal{N}, N(\underline{F}) \subset \underline{F}\}.$$

Theorem

1. *The projection $\psi\colon (\underline{F}, N) \mapsto \underline{F}$ makes $\tilde{\mathcal{N}}$ into a vector bundle over X (in fact, $\tilde{\mathcal{N}} \cong T^*X$).*

The Springer resolution

The nilcone $\mathcal{N}$ is very singular, but it has a resolution of singularities. Let

$$\tilde{\mathcal{N}} = \{(\underline{F}, N) | \underline{F} \in X, N \in \mathcal{N}, N(\underline{F}) \subset \underline{F}\}.$$

Theorem

1. *The projection $\psi: (\underline{F}, N) \mapsto \underline{F}$ makes $\tilde{\mathcal{N}}$ into a vector bundle over X (in fact, $\tilde{\mathcal{N}} \cong T^*X$).*
2. *The projection $\pi: (\underline{F}, N) \mapsto N$ is a resolution of singularities.*

The Springer resolution

The nilcone $\mathcal{N}$ is very singular, but it has a resolution of singularities. Let

$$\tilde{\mathcal{N}} = \{(\underline{F}, N) | \underline{F} \in X, N \in \mathcal{N}, N(\underline{F}) \subset \underline{F}\}.$$

Theorem

1. *The projection* $\psi \colon (\underline{F}, N) \mapsto \underline{F}$ *makes* $\tilde{\mathcal{N}}$ *into a vector bundle over* X *(in fact,* $\tilde{\mathcal{N}} \cong T^*X$*).*
2. *The projection* $\pi \colon (\underline{F}, N) \mapsto N$ *is a resolution of singularities.*

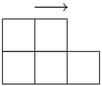
Note that $\pi^{-1}(N) \cong X_N$.

Young diagrams

For any nilpotent, the sizes of the Jordan blocks form a partition, which we can represent as a Young diagram. The number of rows is the number of Jordan blocks.

Young diagrams

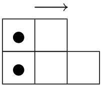
For any nilpotent, the sizes of the Jordan blocks form a partition, which we can represent as a Young diagram. The number of rows is the number of Jordan blocks.



We can think of each box in this diagram as a basis vector, and the nilpotent map sending each box to its right.

Young diagrams

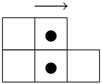
For any nilpotent, the sizes of the Jordan blocks form a partition, which we can represent as a Young diagram. The number of rows is the number of Jordan blocks.



We can think of each box in this diagram as a basis vector, and the nilpotent map sending each box to its right.

Young diagrams

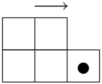
For any nilpotent, the sizes of the Jordan blocks form a partition, which we can represent as a Young diagram. The number of rows is the number of Jordan blocks.



We can think of each box in this diagram as a basis vector, and the nilpotent map sending each box to its right.

Young diagrams

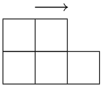
For any nilpotent, the sizes of the Jordan blocks form a partition, which we can represent as a Young diagram. The number of rows is the number of Jordan blocks.



We can think of each box in this diagram as a basis vector, and the nilpotent map sending each box to its right.

Young diagrams

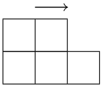
For any nilpotent, the sizes of the Jordan blocks form a partition, which we can represent as a Young diagram. The number of rows is the number of Jordan blocks.



We can think of each box in this diagram as a basis vector, and the nilpotent map sending each box to its right.

Young diagrams

For any nilpotent, the sizes of the Jordan blocks form a partition, which we can represent as a Young diagram. The number of rows is the number of Jordan blocks.



We can think of each box in this diagram as a basis vector, and the nilpotent map sending each box to its right.

By a general result of Spaltenstein, the components $\mathcal{X}_S$ of X_N are in bijection with standard Young tableaux of the same shape.

A **standard tableau** on a diagram is a filling of each box with the numbers $[1, n]$, such that both columns and rows are strictly increasing.

Cup diagrams

For now on, we'll restrict to 2 row diagrams of shape $(n-k,k)$.

Proposition

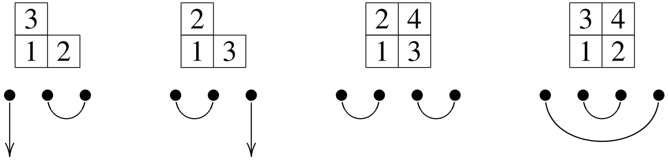
There is a bijection from standard Young tableau of shape $(n-k,k)$, to noncrossing cup diagrams on n points with k cups and $n-2k$ rays, with the right ends of cups being the top row.

Cup diagrams

For now on, we'll restrict to 2 row diagrams of shape $(n-k, k)$.

Proposition

There is a bijection from standard Young tableau of shape $(n-k, k)$, to noncrossing cup diagrams on n points with k cups and $n-2k$ rays, with the right ends of cups being the top row.

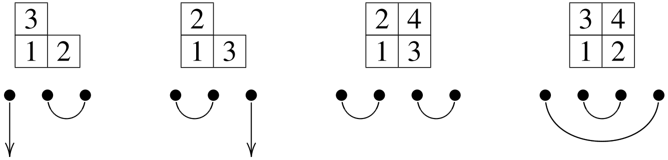


Cup diagrams

For now on, we'll restrict to 2 row diagrams of shape $(n-k,k)$.

Proposition

There is a bijection from standard Young tableau of shape $(n-k,k)$, to noncrossing cup diagrams on n points with k cups and $n-2k$ rays, with the right ends of cups being the top row.



If i is in the bottom row of the tableaux, let $\sigma(i)$ be the other end of the cup, and let $\delta(i) = (\sigma(i) - i + 1)/2$.
That is, $\delta(i)$ is the number of cups "nested inside" the one from i to $\sigma(i)$.

Components of X_N

In the case of shape (k, k), these components can be made very explicit:

Components of X_N

In the case of shape (k, k), these components can be made very explicit:

Proposition (Fung, 2003)

The flag $\mathcal{F}$ lies in the component $\mathcal{X}_S$ if and only if for all i,

$$N^{\delta(i)}(F_{\sigma_S(i)}) = F_{i-1}.$$

Components of X_N

In the case of shape (k,k), these components can be made very explicit:

Proposition (Fung, 2003)

The flag $\mathcal{F}$ lies in the component $\mathcal{X}_S$ if and only if for all i,

$$N^{\delta(i)}(F_{\sigma_S(i)}) = F_{i-1}.$$

For example, when $n = 4, k = 2$:

$$\mathcal{X}_{\begin{smallmatrix}3&4\\1&2\end{smallmatrix}} = \{F_1 \subset F_2 \subset F_3 \subset \mathbb{C}^4 | N(F_3) = F_1\}$$

$$\mathcal{X}_{\begin{smallmatrix}2&4\\1&3\end{smallmatrix}} = \{F_1 \subset F_2 \subset F_3 \subset \mathbb{C}^4 | N(F_2) = \{0\}\}$$

The structure of components

Proposition (Fung)

$\mathcal{X}_S$ is an iterated $\mathbb{P}^1$ bundle of dimension k. In particular, $\dim H^*(\mathcal{X}_S) = 2^k$.

The structure of components

Proposition (Fung)

$\mathcal{X}_S$ is an iterated $\mathbb{P}^1$ bundle of dimension k. In particular, $\dim H^*(\mathcal{X}_S) = 2^k$.

Proof.

Let's choose our spaces one at a time.

If i is in the bottom row, there are no restrictions coming from smaller subspaces, so we get to pick any line in $N^{-1}(V_{i-1})/V_{i-1}$. Since $\dim N^{-1}(V_i)/V_i = 2$, so we get a $\mathbb{P}^1$.

If i is in the top row, $V_i = N^{-\delta(i)}(V_{\sigma^{-1}(i)-1})$, so we only have one choice, and the structure doesn't change.

Everything is in even degree, so the Serre spectral sequence has no differentials, and $\dim H^*(\mathcal{X}_S) = 2^k$. □

Cohomology of components

We let V_i denote the tautological i-dimensional vector bundle on X, and let $R \cong \mathbb{C}[x_1, \ldots, x_n]$, and let e_i be the elementary symmetric polynomials in R.

We have maps given by taking Chern classes $(c_1(V_i/V_{i-1}) \mapsto x_i)$, and getting "obvious" relations coming from K-theory.

$$X \hookleftarrow X_N \hookleftarrow \mathcal{X}_S$$

Cohomology of components

We let V_i denote the tautological i-dimensional vector bundle on X, and let $R \cong \mathbb{C}[x_1, \ldots, x_n]$, and let e_i be the elementary symmetric polynomials in R.

We have maps given by taking Chern classes $(c_1(V_i/V_{i-1}) \mapsto x_i)$, and getting "obvious" relations coming from K-theory.

$$H^*(X) \longrightarrow H^*(X_N) \longrightarrow H^*(\mathcal{X}_S)$$

Cohomology of components

We let V_i denote the tautological i-dimensional vector bundle on X, and let $R \cong \mathbb{C}[x_1, \ldots, x_n]$, and let e_i be the elementary symmetric polynomials in R.

We have maps given by taking Chern classes $(c_1(V_i/V_{i-1}) \mapsto x_i)$, and getting "obvious" relations coming from K-theory.

$$\begin{array}{ccccc} H^*(X) & \longrightarrow & H^*(X_N) & \longrightarrow & H^*(\mathcal{X}_S) \\ \uparrow & & \uparrow & & \\ R/(e_i) & \longrightarrow & R/(x_i^2, e_i) & & \end{array}$$

"Obvious" relations

Using the equations of a component, we have maps

$$\begin{array}{ccc}
V_{\sigma(i)} & \twoheadrightarrow & N^{-1}(V_{i-1}) \\
\uparrow & & \uparrow \\
V_{\sigma(i)-1} & \twoheadrightarrow & V_i \\
\uparrow & & \uparrow \\
N(V_{\sigma(i)}) & \twoheadrightarrow & V_{i-1}
\end{array}$$

We have an exact sequence $0 \to L_i \to J \to L_{\sigma(i)} \to 0$ where J is trivial in K-theory, so $\mathcal{O}_{\mathcal{X}_S} \cong L_i \otimes L_{\sigma(i)}$.

In Chern classes, this says $x_i + x_{\sigma(i)} = 0$ in $H^*(\mathcal{X}_S)$.

Cohomology of components

We let V_i denote the tautological i-dimensional vector bundle on X, and let $R \cong \mathbb{C}[x_1, \ldots, x_n]$, and let e_i be the elementary symmetric polynomials in R.

We have maps given by taking Chern classes $(c_1(V_i/V_{i-1}) \mapsto x_i)$, and getting "obvious" relations coming from K-theory.

$$\begin{array}{ccccc} H^*(X) & \longrightarrow & H^*(X_N) & \longrightarrow & H^*(\mathcal{X}_S) \\ \uparrow & & \uparrow & & \uparrow \\ R/(e_i) & \longrightarrow & R/(x_i^2, e_i) & \longrightarrow & R/(x_i + x_{\sigma(i)}, x_i^2) \end{array}$$

Cohomology of components

We let V_i denote the tautological i-dimensional vector bundle on X, and let $R \cong \mathbb{C}[x_1, \ldots, x_n]$, and let e_i be the elementary symmetric polynomials in R.

We have maps given by taking Chern classes $(c_1(V_i/V_{i-1}) \mapsto x_i)$, and getting "obvious" relations coming from K-theory.

$$\begin{array}{ccccc} H^*(X) & \longrightarrow & H^*(X_N) & \longrightarrow & H^*(\mathcal{X}_S) \\ \uparrow & & \uparrow & & \uparrow \\ R/(e_i) & \longrightarrow & R/(x_i^2, e_i) & \longrightarrow & R/(x_i + x_{\sigma(i)}, x_i^2) \end{array}$$

The right hand map is an isomorphism for dimensional reasons.

Intersections of components

Proposition

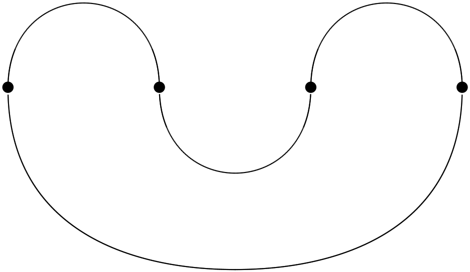
The intersections of components are also iterated $\mathbb{P}^1$-bundles, with the dimension of $\mathcal{X}_S \cap \mathcal{X}_{S'}$ given by the number of circles in the union of the cup diagrams.

Intersections of components

Proposition

The intersections of components are also iterated $\mathbb{P}^1$-bundles, with the dimension of $\mathcal{X}_S \cap \mathcal{X}_{S'}$ given by the number of circles in the union of the cup diagrams.

$$\mathcal{X}_{\begin{smallmatrix}3&4\\1&2\end{smallmatrix}} \cap \mathcal{X}_{\begin{smallmatrix}2&4\\1&3\end{smallmatrix}} \cong \mathbb{P}^1$$



Cohomology of intersections.

Proposition

$R/(x_i + x_{\sigma_S(i)}, x_i + x_{\sigma_{S'}(i)}, x_i^2) \cong H^*(\mathcal{X}_S \cap \mathcal{X}_{S'})$

Cohomology of intersections.

Proposition

$R/(x_i + x_{\sigma_S(i)}, x_i + x_{\sigma_{S'}(i)}, x_i^2) \cong H^*(\mathcal{X}_S \cap \mathcal{X}_{S'})$

Proof.

We know we have a map. It's onto in H^2 and thus onto. The dimension of both sides is $2^{\#\text{ of circles}}$, so it's an isomorphism. □

Cohomology of intersections.

Proposition

$R/(x_i + x_{\sigma_S(i)}, x_i + x_{\sigma_{S'}(i)}, x_i^2) \cong H^*(\mathcal{X}_S \cap \mathcal{X}_{S'})$

Proof.

We know we have a map. It's onto in H^2 and thus onto. The dimension of both sides is $2^{\#\text{ of circles}}$, so it's an isomorphism. □

This tells us about the bimodule multiplication

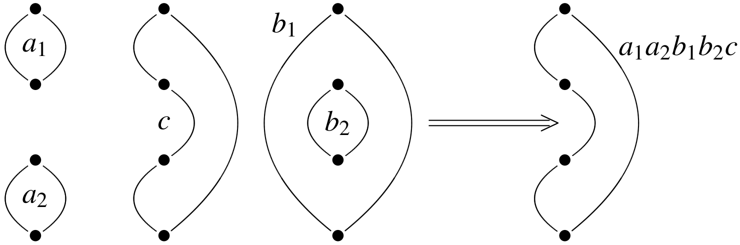
$$H^*(\mathcal{X}_S) \otimes H^*(\mathcal{X}_S \cap \mathcal{X}_{S'}) \otimes H^*(\mathcal{X}_{S'}) \to H^*(\mathcal{X}_S \cap \mathcal{X}_{S'}).$$

Combinatorial multiplication

We can describe this multiplication combinatorially using cobordisms.

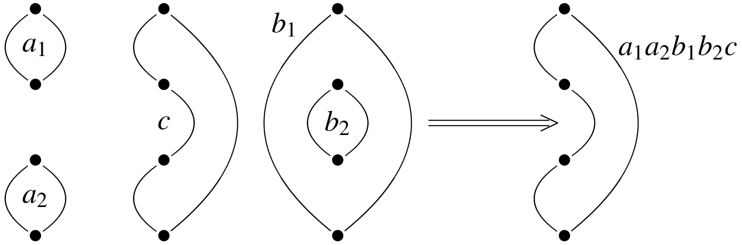
Combinatorial multiplication

We can describe this multiplication combinatorially using cobordisms.



Combinatorial multiplication

We can describe this multiplication combinatorially using cobordisms.



Maybe some of you have seen this sort of multiplication before: it appears in Khovanov's algebra $\mathcal{H}^k$.

Convolution

Let $\tilde{\mathcal{X}} = \bigsqcup \mathcal{X}_S$ be the disjoint union of all components of X_N.

Convolution

Let $\tilde{\mathcal{X}} = \bigsqcup \mathcal{X}_S$ be the disjoint union of all components of X_N.

Theorem (Stroppel-W.)

Our combinatorial picture with circles gives an isomorphism of vector spaces

$$H^*(\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}}) \cong \oplus_{S,S'} H^*(\mathcal{X}_S \cap \mathcal{X}_{S'}) \cong \mathcal{H}^k.$$

Convolution

Let $\tilde{\mathcal{X}} = \bigsqcup \mathcal{X}_S$ be the disjoint union of all components of X_N.

Theorem (Stroppel-W.)

Our combinatorial picture with circles gives an isomorphism of vector spaces

$$H^*(\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}}) \cong \oplus_{S,S'} H^*(\mathcal{X}_S \cap \mathcal{X}_{S'}) \cong \mathcal{H}^k.$$

The convolution product on cohomology makes $H^*(\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}})$ into an algebra, given by pull, cap and push on the diagram

$$\begin{array}{ccccc}
\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} & \xleftarrow{p_{12}} & & & \\
 & & \tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} & \xrightarrow{p_{13}} & \tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} \\
\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} & \xleftarrow{p_{23}} & & &
\end{array}$$

Convolution

Let $\tilde{\mathcal{X}} = \bigsqcup \mathcal{X}_S$ be the disjoint union of all components of X_N.

Theorem (Stroppel-W.)

Our combinatorial picture with circles gives an isomorphism of vector spaces

$$H^*(\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}}) \cong \oplus_{S,S'} H^*(\mathcal{X}_S \cap \mathcal{X}_{S'}) \cong \mathcal{H}^k.$$

The convolution product on cohomology makes $H^*(\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}})$ into an algebra, given by pull, cap and push on the diagram

$$\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} \xleftarrow{p_{12}} \tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} \xrightarrow{p_{13}} \tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}}$$

$$\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}} \xrightarrow{p_{23}} \tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}}$$

This doesn't seem to be an algebra isomorphism! But it's very close!

Our algebra vs. Khovanov's

Let ω be a parameter satisfying $\omega^4 = 1$.

Theorem

There exists an algebra A^k_ω such that

$$A^k_{\pm 1} \cong \mathcal{H}^k \qquad A^k_{\pm i} \cong H^*(\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}})$$

Our algebra vs. Khovanov's

Let ω be a parameter satisfying $\omega^4 = 1$.

Theorem

There exists an algebra A^k_ω such that

$$A^k_{\pm 1} \cong \mathcal{H}^k \qquad A^k_{\pm i} \cong H^*(\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}})$$

This algebra is based on a modified TQFT which distinguishes between the Morse moves creating two circles from one which result in nested or unnested circles.

Our algebra vs. Khovanov's

Let ω be a parameter satisfying $\omega^4 = 1$.

Theorem

There exists an algebra A^k_ω such that

$$A^k_{\pm 1} \cong \mathcal{H}^k \qquad A^k_{\pm i} \cong H^*(\tilde{\mathcal{X}} \times_{X_N} \tilde{\mathcal{X}})$$

This algebra is based on a modified TQFT which distinguishes between the Morse moves creating two circles from one which result in nested or unnested circles.

Proposition

This algebra/modified TQFT can be used to construct a knot homology theory Kh_ω which reduces to classical Khovanov homology when $\omega = \pm 1$.

Functoriality?

It's not so upsetting that our algebra differs from Khovanov's in the small matter of signs, since *the signs in classical Khovanov homology are wrong.* The theory is only functorial up to sign.

Functoriality?

It's not so upsetting that our algebra differs from Khovanov's in the small matter of signs, since *the signs in classical Khovanov homology are wrong.* The theory is only functorial up to sign.

Morrison and Walker have already constructed a knot homology theory depending on the same parameter ω, which is Khovanov homology at $\omega = \pm 1$, and which is honestly functorial at $\omega = \pm i$.

Well, that looks familiar.

Functoriality?

It's not so upsetting that our algebra differs from Khovanov's in the small matter of signs, since *the signs in classical Khovanov homology are wrong.* The theory is only functorial up to sign.

Morrison and Walker have already constructed a knot homology theory depending on the same parameter ω, which is Khovanov homology at $\omega = \pm 1$, and which is honestly functorial at $\omega = \pm i$.

Well, that looks familiar.

Question

Is Kh_ω honestly functorial for $\omega = \pm i$? Is it the same as Morrison and Walker's theory? I would suspect so, but I haven't found the isomorphism yet.

Generalizations

There are a large number of varieties with similar structures to Springer fibers, including quiver varieties, hypertoric varieties, and other hyperkähler quotients (more generally, symplectic singularities).

One of the most easily studied such is hypertoric varieties. These are "quaternionic analogues" of toric varieties.

Generalizations

There are a large number of varieties with similar structures to Springer fibers, including quiver varieties, hypertoric varieties, and other hyperkähler quotients (more generally, symplectic singularities).

One of the most easily studied such is hypertoric varieties. These are "quaternionic analogues" of toric varieties.

Alternatively, you can think of them as a version of the cotangent bundle to a toric variety with some extra strata added that remember the original toric variety was a quotient by a non-free action.

Hypertoric varieties

Whereas toric varieties remember a polytope, hypertoric varieties remember a hyperplane arrangement (with the toric varieties corresponding to the chambers of the arrangement appearing).

Hypertoric varieties

Whereas toric varieties remember a polytope, hypertoric varieties remember a hyperplane arrangement (with the toric varieties corresponding to the chambers of the arrangement appearing).

Theorem (Braden-Licata-Proudfoot-W.)

The corresponding algebras for hypertoric varieties can be explicitly described in terms of a quiver presentation (in fact, as quotient of the Deligne groupoid algebra of the hyperplane arrangement.

Hypertoric varieties

Whereas toric varieties remember a polytope, hypertoric varieties remember a hyperplane arrangement (with the toric varieties corresponding to the chambers of the arrangement appearing).

Theorem (Braden-Licata-Proudfoot-W.)

The corresponding algebras for hypertoric varieties can be explicitly described in terms of a quiver presentation (in fact, as quotient of the Deligne groupoid algebra of the hyperplane arrangement.

Furthermore, hyperplane arrangements (and thus hypertoric varieties) have a notion of duality. The geometry of this duality still rather mysterious.

Theorem (BLPW)

The algebras associated to Gale dual hypertoric varieties are Koszul dual.

Outlook

There's a lot more to explore here. Possible connections include:

Outlook

There's a lot more to explore here. Possible connections include:

1. symplectic duality (Braden, Licata & Proudfoot)

Outlook

There's a lot more to explore here. Possible connections include:

1. symplectic duality (Braden, Licata & Proudfoot)
2. quantization of symplectic varieties (Kaledin & Bezrukavnikov)

Outlook

There's a lot more to explore here. Possible connections include:

1. symplectic duality (Braden, Licata & Proudfoot)
2. quantization of symplectic varieties (Kaledin & Bezrukavnikov)
3. the Fukaya category (A-model) of hyperkähler quotients (Witten, etc.)

Outlook

There's a lot more to explore here. Possible connections include:

1. symplectic duality (Braden, Licata & Proudfoot)
2. quantization of symplectic varieties (Kaledin & Bezrukavnikov)
3. the Fukaya category (A-model) of hyperkähler quotients (Witten, etc.)
4. derived categories of coherent sheaves (B-model)

Outlook

There's a lot more to explore here. Possible connections include:

1. symplectic duality (Braden, Licata & Proudfoot)
2. quantization of symplectic varieties (Kaledin & Bezrukavnikov)
3. the Fukaya category (A-model) of hyperkähler quotients (Witten, etc.)
4. derived categories of coherent sheaves (B-model)
5. Koszul duality

Outlook

There's a lot more to explore here. Possible connections include:

1. symplectic duality (Braden, Licata & Proudfoot)
2. quantization of symplectic varieties (Kaledin & Bezrukavnikov)
3. the Fukaya category (A-model) of hyperkähler quotients (Witten, etc.)
4. derived categories of coherent sheaves (B-model)
5. Koszul duality
6. mirror symmetry of 6-D gauge theories

Outlook

There's a lot more to explore here. Possible connections include:

1. symplectic duality (Braden, Licata & Proudfoot)
2. quantization of symplectic varieties (Kaledin & Bezrukavnikov)
3. the Fukaya category (A-model) of hyperkähler quotients (Witten, etc.)
4. derived categories of coherent sheaves (B-model)
5. Koszul duality
6. mirror symmetry of 6-D gauge theories
7. representation theory of finite W-algebras

Thanks, y'all.