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Derandomizing the AW matrix-valued Chernoff bound using pessimistic estimators and applications

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Abstract

Ahlswede and Winter [AW02] introduced a Chernoff bound for matrix-valued random variables, which is a non-trivial generalization of the usual Chernoff bound for real-valued random variables. We present an efficient derandomization of their bound using the method of pessimistic estimators (see Raghavan [Rag88]). As a consequence, we derandomize a construction of Alon and Roichman [AR94] (see also [LR04, LS04]) to efficiently construct an expanding Cayley graph of logarithmic degree on any (possibly non-abelian) group. This also gives an optimal solution to the homomorphism testing problem of Shpilka and Wigderson [SW04]. We also apply these pessimistic estimators to the problem of solving semi-definite covering problems, thus giving a deterministic algorithm for the quantum hypergraph cover problem of [AW02].

The results above appear as theorems in the paper [WX05a] (see also [WX05b]), as consequences to the main theorem of that paper: a randomness efficient sampler for matrix valued functions via expander walks. However, we discovered an error in the proof of that main theorem (which we briefly describe in the appendix). That main theorem stating that the expander walk sampler is good for matrix-valued functions thus remains open. One purpose of the current paper is to show that the applications in that paper hold true despite our inability to prove the expander walk sampler theorem for matrix-valued functions.

1 Introduction

Chernoff bounds are extremely useful throughout theoretical computer science. Intuitively, they say that a random sample approximates the average, with probability of deviation that goes down exponentially with the number of samples. Typically we are concerned with real-valued random variables, but recently several applications have called for matrix-valued random variables. Such a bound was given by Ahlswede and Winter [AW02].

In particular, the matrix-valued bound seems useful in giving new proofs of probabilistic constructions of expander graphs [AR94] and also in the randomized rounding of semi-definite covering problems, with further applications in quantum information theory [AW02].

In this paper we use the method of pessimistic estimators, originally formulated in [Rag88][1], to derandomize the Chernoff bound of [AW02], and in the process derandomize the Alon-Roichman theorem and the randomized rounding of covering SDP's.

The results of this paper prove the claimed applications of our previous paper [WX05a], and in fact supersede them in simplicity and efficiency. We regret to inform the community that

[1] The simpler method of conditional probabilities was described earlier in the first edition of [Spe94].

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