Derandomizing the Ahlswede-Winter matrix-valued Chernoff bound using pessimistic estimators, and applications

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Abstract: Ahlswede and Winter [IEEE Trans. Inf. Th. 2002] introduced a Chernoff bound for matrix-valued random variables, which is a non-trivial generalization of the usual Chernoff bound for real-valued random variables. We present an efficient derandomization of their bound using the method of pessimistic estimators (see Raghavan [JCSS 1988]). As a consequence, we derandomize an efficient construction by Alon and Roichman [RSA 1994] of an expanding Cayley graph of logarithmic degree on any (possibly non-abelian) group. This gives an optimal solution to the homomorphism testing problem of Shpilka and Wigderson [STOC 2004]. We also apply these pessimistic estimators to the problem of solving semidefinite covering problems, thus giving a deterministic algorithm for the quantum hypergraph cover problem of Ahslwede and Winter.

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simistic estimators

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