

Entropy Waves, The Zig-Zag Graph Product, and New Constant-Degree Expanders

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Abstract

The main contribution of this work is a new type of graph product, which we call the **zig-zag product**. Taking a product of a large graph with a small graph, the resulting graph inherits (roughly) its size from the large one, its degree from the small one, and its expansion properties from both! Iteration yields simple explicit constructions of constant-degree expanders of arbitrary size, starting from one constant-size expander.

Crucial to our intuition (and simple analysis) of the properties of this graph product is the view of expanders as functions which act as “entropy wave” propagators — they transform probability distributions in which entropy is concentrated in one area to distributions where that concentration is dissipated. In these terms, the graph product affords the constructive interference of two such waves.

Subsequent work [ALW01, MW01] relates the zig-zag product of graphs to the standard semidirect product of groups, leading to new results and constructions on expanding Cayley graphs.

Keywords: expander graphs, graph products, entropy

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