

The Parallel Complexity of Element Distinctness is $\Omega(\sqrt{\log n})$

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Abstract

We consider the problem of element distinctness. Here n synchronized processors, each given an integer input, must decide whether these integers are pairwise distinct, while communicating via an infinitely large shared memory.

If simultaneous write access to a memory cell is forbidden, then a lower bound of $\Omega(\log n)$ on the number of steps easily follows (from S. Cook, C. Dwork, and R. Reischuk, *SIAM J. Comput.*, 15 (1986), pp. 87-97). When several (different) values can be written simultaneously to any cell, then there is a simple algorithm requiring $O(1)$ steps.

We consider the intermediate model, in which simultaneous writes to a single cell are allowed only if all values written are equal. We prove a lower bound of $\Omega((\log n)^{1/2})$ steps, improving the previous lower bound of $\Omega(\log \log \log n)$ steps (F. E. Fich, F. Meyer auf der Heide, and A. Wigderson, *Adv. In Comput.*, 4(1987), pp. 1—15).

The proof uses Ramsey-theoretic and combinatorial arguments. The result implies a separation between the powers of some variants of the PRAM model of parallel computation.