A STRONG DIRECT PRODUCT THEOREM FOR CORRUPTION AND THE MULTIPARTY COMMUNICATION COMPLEXITY OF DISJOINTNESS

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Abstract. We prove that two-party randomized communication complexity satisfies a strong direct product property, so long as the communication lower bound is proved by a "corruption" or "one-sided discrepancy" method over a rectangular distribution. We use this to prove new $n^{\Omega(1)}$ lower bounds for 3-player number-on-the-forehead protocols in which the first player speaks once and then the other two players proceed arbitrarily. Using other techniques, we also establish an $\Omega(n^{1/(k-1)}/(k-1))$ lower bound for k-player randomized number-on-the-forehead protocols for the disjointness function in which all messages are broadcast simultaneously. A simple corollary of this is that general randomized number-on-the-forehead protocols require $\Omega(\log n/(k-1))$ bits of communication to compute the disjointness function.

Keywords. Communication complexity, direct product, direct sum, multiparty protocols, lower bounds.

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1. Introduction

1.1. Number-on-the-forehead communication protocols. A fundamental problem in communication complexity is understanding the amount of communication necessary to compute the two-player disjointness function: Alice and Bob are each given a subset of $\{1, \ldots n\}$ and they want to determine whether or not they share a common element (Babai, Frankl & Simon 1986; Kalyanasundaram & Schnitger 1987; Raz & Wigderson 1992; Razborov 1992). A natural extension of two-party disjointness is k-party disjointness. In this