2007 Course Descriptions

Beginning Lecture Course

Lecturers: Amy Ksir, United States Naval Academy

Jessica Sidman, Mount Holyoke College

Teaching Assistants: Diane Davis, Milena Hering

Titles for the Beginning Lectures Series:

Amy Ksir, "Enumerative Geometry and String Theory"

Jessica Sidman, "Toric Varieties"

Beginning Lecture Course Suggested Background:

Amy Ksir, "Enumerative geometry and string theory"

Enumerative geometry is the art of counting geometric objects with specified properties. These kinds of problems were posed by the ancient Greeks, and are still the subject of active research today. In particular, they have enjoyed a renaissance due to a recent influx of ideas from string theory.

We will begin by discussing Steiner's problem, which was first solved in

1859: How many plane conics are tangent to five given conics? The solution to this problem will introduce many of the key tools in enumerative geometry, and in algebraic geometry in general. We will then do a brief tour of the related areas of physics and explore the connections between string theory and enumerative geometry.

Possible topics:

Bezout's Theorem

Excess intersection

Duality

Blowing up

Classical mechanics

Calabi-Yau manifolds

Gromov-Witten invariants

Quantum cohomology

Prerequisites: A course in linear algebra and the ability to write proofs.

Any of the following will also be helpful, but none are necessary: An abstract algebra course including rings and ideals; real analysis, complex analysis, or topology; first-year physics.

Suggested references:

Sheldon Katz, Enumerative geometry and string theory

Jessica Sidman - "Toric varieties"

An algebraic variety is the solution set of a system of polynomial equations. In this short course we will focus on a special class of varieties called toric varieties. A toric variety is the solution set of a system of binomial equations. Many important classical examples of varieties are toric, and toric varieties appear in surprising places -- including statistics and the physics of string theory!

We will briefly discuss the correspondence between ideals and varieties.

The goal of the course is to introduce students to concrete examples of varieties and to explore how to work with them. We will work with toric varieties by hand and using computers. We will discuss applications if time permits.

Possible topics:

Varieties in projective space

The Nullstellensatz

Rational normal curves

Rational normal scrolls

Segre products

Veronese varieties

Blowups

Groebner bases

Algebraic statistics

Background: Course work in linear algebra is essential. Students should know the definition of the nullspace (kernel) of a matrix and how to compute a basis for it. Some knowledge of abstract algebra will be helpful, especially familiarity with either quotient rings or quotient groups.

Suggested references

David Cox, John Little, Donal O'Shea, Ideals, Varieties, and Algorithms.

David S. Dummit and Richard R. Foote, Abstract Algebra

Rekha R. Thomas, Lectures in Geometric Combinatorics.

Advanced Lecture Course

Lecturers: Frances Kirwan, Oxford University

V. Lakshmibai, Northeastern Univeristy

Teaching Assistants: Wei Ho, Lauren Williams

Titles for the Graduate Lectures Series:

Frances Kirwan, "Quotients of algebraic varieties by group actions"

V. Lakshmibai, "Flag Varieties"

Advanced Lecture Course Suggested Background:

V. Lakshmibai, "Flag Varieties"

This course will focus on Flag Varieties & their Schubert varieties - their geometric & representation-theoretic aspects. After introducing the Grassmann varieties, we shall present the classical work of Hodge on the Grassmannian giving a basis for the homogeneous co-ordinate ring of the Grassmannian (as well as its Schubert varieties) for the Plucker embedding in terms of "standard monomials" in the Plucker co-ordinates.

We shall then show how these results extend to the flag varieties & their Schubert varieties. We shall also discuss some important algebraic varieties related to Schubert varieties.

LIST OF POSSIBLE TOPICS:

Grassmannians & their Schubert varieties

Hodge's work on the Grassmannian Flag Varieties & their Schubert varieties

Generalization of Hodge's work to flag Varieties Singular loci of Schubert varieties

Some affine varieties related to Schubert varieties

Representation-theoretic aspects of Flag Varieties

SUGGESTED BACKGROUND:

Familiarity with commutative algebra - rings, ideals, modules, fields etc

Familiarity with basics of algebraic geometry - affine & projective varieties, their co-ordinate rings etc

Some familiarity with representation theory, especially, representation theory of GL(n) is useful but not required.

SUGGESTED REFERENCES:

REFERENCES FOR ALGEBRAIC GEOMETRY:

1. D. Eisenbud and J. Harris, The geometry of schemes, Grad. Texts in Math # 197, Springer.

2. R. Hartshorne, Algebraic geometry, Grad. Texts in Mathematics, 52.

3. D. Mumford, The red book of varieties and schemes, Lecture Notes in Math, 1358.

REFERENCES FOR ALGEBRA:

1. D. Eisenbud, Commutative algebra with a view to algebraic geometry.

2. H. Matsumura, Commutative Ring Theory

REFERENCES FOR REPRESENTATION THEORY:

1. W. Fulton and J. Harris, Representation theory, A first course, Grad. Texts in Math # 129, Springer-Verlog.

2. James Humphreys, Introduction to Lie algebras and representation theory, Grad. Texts in Math # 21, Springer-Verlog.

Frances Kirwan, "Quotients of algebraic varieties by group actions"

These lectures will study the construction of quotients of algebraic varieties by algebraic group actions. As motivation, we will begin by discussing how the concept of a moduli space arises in classification problems in algebraic geometry. We will see that (when they exist) moduli spaces can often be constructed as quotients of algebraic varieties by group actions, but that only very special algebraic group actions on algebraic varieties have quotients which are themselves algebraic varieties in a natural way.

Given a suitable reductive group action on a variety X, Mumford's geometric invariant theory (GIT) provides us with open subvarieties of X with well behaved quotient varieties. We will look at some simple examples of this construction, as well as some applications to moduli spaces.

Next we will study the link between GIT and the concept of reduction in symplectic geometry using moment(um) maps, which leads to methods for studying the topology of GIT quotients and moduli spaces in complex algebraic geometry.

Finally we will discuss briefly what happens when we try to construct quotients of algebraic varieties by actions of algebraic groups which are not reductive.

PREREQUISITES: Undergraduate algebra including group actions on sets.

Undergraduate topology.

Some basic algebraic geometry including affine and projective varieties and their coordinate rings.

Some basic differential geometry including manifolds and their tangent bundles.

Some familiarity with the theory of Lie groups and Lie algebras would be useful but not essential.

SUGGESTED REFERENCES:

SUGGESTED BOOKS FOR PREREQUISITES:

K Smith et al, An invitation to algebraic geometry, Springer 2000

G Kempf, Algebraic varieties, London Math Soc Lecture Notes 172, CUP 1993

R Hartshorne, Algebraic geometry (Chapter 1 only), Springer GTM 52.

D Huybrechts, Complex geometry: an introduction, Springer 2004

D Bump, Lie groups, GTM 225, Springer 2004

R Bott and L Tu, Differential forms in algebraic topology, Springer

BOOKS FOR THE COURSE:

Most useful (especially the first):

S Mukai, An introduction to invariants and moduli, Cambridge Studies in Advanced Mathematics 81, CUP 2003 P Newstead,Introduction to moduli problems and orbit spaces, Tata Institute Notes, Springer 1978.

D Mumford, J Fogarty, F Kirwan, Geometric invariant theory (3rd edition), Springer

Also useful (in addition to the suggested books for prerequisites):

A Borel, Linear algebraic groups, GTM 126, Springer

Tauvel and Yu, Lie algebras and algebraic groups, Springer

J Harris and D Morrison, Moduli of curves, Springer

V S Varadarajan, Lie groups, Lie algebras and their representations, GTM 98, Springer 1984

Walter Ferrer Santos and Alvaro Rittatore, Actions and invariants of algebraic groups,

Chapman and Hall, 2005 A Bialynicki-Birula et al, Invariant theory, Encyclopedia of Mathematical Sciences 131, Springer 2002