2006 Course Descriptions

Beginning Lecture Course

Lecturers: Giuliana Davidoff, Mount Holyoke College

Margaret Robinson, Mount Holyoke College

Teaching Assistants: Amanda Folsom, UCLA
Cornelia Yuen, University of Michigan

Our course will begin with the definition of the Riemann zeta function, its Euler product expansion, and its analytic properties. We will generalize the classical zeta function to other types such as the Dirichlet L-function and L-functions associated with some algebraic number fields. Using the Riemann zeta function as a prototype, we will move on to zeta functions associated with polynomials defined over finite fields (Hasse-Weil zeta function) and those associated with polynomials defined over p-adic local fields (Igusa local zeta functions).
List of possible topics:

- Riemann zeta function: meromorphic on the complex plane, Dirichlet expansion, Euler product expansion, functional equation, and poles, residues and zeros
- Dirichlet L-functions
- Fundamental properties of finite fields and polynomials over finite fields
- Multplicative characters
- Weil zeta function for curves over finite fields
- p-adic numbers and p-adic analysis
- Igusa Local zeta function

Beginning Lecture Course Suggested Background

Group, ring and field theory as covered in most undergraduate abstract algebra classes, real analysis at the undergraduate level, some undergraduate complex analysis and number theory would help too.

Advanced Lecture Course

Lecturers: 1st week - Kate Okikiolu, University of California-San Diego

2nd week - Audrey Terras, University of California-San Diego

Teaching Assistants: Amanda Beeson, UCSD
Brooke Feigon, UCLA
Ruth Gornet, University of Texas at Arlington

Week 1

Title: **Spectral Zeta Functions in Geometry**

Lectures 1 and 2.

For a non-negative elliptic differential operator L defined on a smooth compact manifold, the spectrum forms a discrete sequence tending to infinity. We define the Minakshisundaram-Pleijel zeta function and local zeta function associated to L, and by constructing the powers of L we show that these zeta functions have analytic continuations to meromorphic functions on the plane. We express the residues at the poles and some special values in terms of the Guillemin-Wodzicki residue, and see cases when this vanishes.

Lecture 3.

We discuss the analytic torsion, a topological invariant whose definition was one of the first applications of spectral zeta functions.

We will also discuss applications of spectral zeta functions to geometry and open problems.

Pre-requisites: The course will assume some Real and complex analysis and it will be helpful to be familiar with smooth manifolds and a little algebraic topology.

Specifically, analytic prerequisites are Cauchy's integral formula, properties of the Fourier transform, partitions of unity and interchange of limits - for example dominated convergence theorem and Fubini's theorem.

Week 2

Title: **Zeta and L-Functions of Graphs**

Lectures 1 and 2.

- Introduction to zeta functions of graphs (history, comparisons with Riemann, Selberg, Ruelle zeta functions)
- Definitions of 3 kinds of graph zeta functions: vertex, edge and path
- Basic properties, examples, Bass proof of Ihara's determinant formula
- Analogs of Riemann Hypothesis, Montgomery-Odlyzko Law
- Graph Theory Prime Number Theorem

Lectures 3 and 4.

- Quick review of basic facts about Artin L-functions of normal extensions of algebraic number fields with examples
- Tchebotarev density theorem, proof sketch
- Galois theory of graph coverings, examples
- Artin L-functions of graphs
- Applications such as factorization of zeta functions, computation of density of paths which split in coverings, generalization of Cayley and Schreier graphs, construction of connected k-regular graphs which are isospectral but not isomorphic

References:

zeta and L-functions of graph coverings

H. M. Stark and A. Terras, Zeta functions of finite graphs and coverings, Advances in Math, 121 (1996), 124-165

H. M. Stark and A. Terras, Zeta functions of finite graphs and coverings, Part II, Advances in Math., 154 (2000), 132-195.

group representations, etc.

Terras, Fourier Analysis on Finite Groups and Applications, Cambridge U. Press, Cambridge, 1999.

number-theoretic zeta functions

H. Davenport, Multiplicative Number Theory, Springer-Verlag, N.Y., 1980

K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, Springer-Verlag, N.Y., 1998.

H. M. Stark, Galois theory, algebraic number theory and zeta functions, in From Number Theory to Physics, M. Waldschmidt et al (Eds.), Springer-Verlag, Berlin, 1992.

zeta functions and quantum chaos

N. Katz and P. Sarnak, Zeroes of zeta functions and symmetry, Bull. Amer. Math. Soc., 36 (1999), 1-26.