2004 Women in Mathematics Program

Monday, May 17, 2004 (All day) - Friday, May 28, 2004 (All day)
(2003-2004)

Analysis and Nonlinear PDEs

May 17 - 28, 2004

Course Descriptions:

Beginning Lecture Course: Harmonic Analysis: from Fourier to Haar
Prerequisites: advanced calculus and linear algebra

Lecturers: 1st week - Lesley Ward (Harvey Mudd College)
2nd week - Cristina Pereyra (University of New Mexico)

Teaching Assistants: Manuela Longoni de Castro (Univ. of New Mexico) for Lesley Ward
Stephanie Molnar (UCLA) for Cristina Pereyra

Outline: This two week course aims to expose the students to the basics of harmonic analysis. It ranges from Fourier's heat equation, and the decomposition of functions into sums of cosines and sines (frequency analysis) to dyadic harmonic analysis (or decomposition into Haar basis functions, involving time localization). In between these two different ways of decomposing functions lies a whole world of time/frequency analysis (wavelets) which we will touch on, although we will concentrate on the Fourier and Haar cases.

In the **first week**, some of the classical results on Fourier series will be discussed. These include different modes of convergence, Gibbs' phenomenon, and the interplay with differentiability, as well as the conceptually simpler case of Fourier analysis on finite dimensional spaces and the celebrated and influential Fast Fourier Transform, an algorithm discovered by Euler almost 200 years ago and rediscovered by Tukey and Cooley in the 1960s.

In the **second week**, the Hilbert transform, probably the most important operator in harmonic analysis after the Fourier transform, will be introduced, as well as its connections to complex and Fourier analysis. The Haar basis and the geometry of dyadic intervals will be analyzed, and we will show how to express the Hilbert transform as a superposition of dyadic operators. Wavelets will be introduced, and the framework of multiresolution analysis will be discussed in terms of the Haar basis. The problem sessions will be oriented toward applications and, sometimes, careful calculations avoided in class. We will use software such as the Matlab Wavelet Toolbox to illustrate the power of these techniques in the "real world".

We will introduce as needed concepts from Hilbert spaces, Banach spaces, and the theory of distributions. In particular we will try to compare finite- and infinite-dimensional spaces, and remark on their differences and their analogies.

Suggested references:

1. "Fourier Analysis - an introduction" by E. Stein & R. Shakarshi. Princeton Lectures in Analysis I, Princeton Univ. Press 2003.
2. "Introduction to Fourier Analysis and Wavelets" by Mark A. Pinsky. The Brooks/Cole Series in Advanced Mathematics, Paul J. Sally, Jr. Editor. Brooks Cole; 1st edition (October 19, 2001). ISBN 0-534-37660-6.
3. "A Panorama of Harmonic Analysis" by S. G. Krantz. The Carus Mathematical Monographs 27, AMS 1999.
4. "An Introduction to Wavelets Through Linear Algebra" by Michael W. Frazier (Undergraduate Texts in Mathematics) Springer Verlag 1999.

Advanced Lecture Course: The wave equation: classical and modern methods

Lecturers: 1st week - Jill Pipher (Brown University)

2nd week - Gigliola Staffilani (MIT)

Teaching Assistants: Juhi Jang (Brown University)

Sarah Raynor (The Fields Institute and the University of Toronto)

Outline: In this two week course we present some classical and more modern methods in the study of the linear and nonlinear wave equations when data are assigned at the initial time. We shall see that abstract functional analysis, harmonic analysis and Fourier analysis can be used to solve a very "Physical" problem. The course will be self-contained and accessible to first year graduate students who know some basic real variable theory (measure theory, Banach and Hilbert spaces, linear operators). No knowledge

of partial differential equations is assumed.

In the **first week** we introduce the homogeneous and inhomogeneous Cauchy problem associated to the linear wave equation, and prove a fundamental property of solutions, the "finite speed of propagation". As preparation for the nonlinear theory, we will discuss a family of a priori estimates for these solutions: the Strichartz estimates. We will prove certain of these estimates, but the others require more sophisticated tools of harmonic analysis and are beyond the scope of this course.

In the **second week** we introduce some nonlinear wave equations and present the classical energy method used to solve the associated Cauchy problem under the assumption that the initial data are smooth enough. Then, an example is presented to show how the Strichartz estimates may be used to relax the smoothness assumption on the data.

We hope that the course will give some flavor of the ideas and tools that have been recently used to make extraordinary progress in the study of wave maps and quasilinear equations.

Materials will be drawn from the following sources: "Harmonic Analysis" by E. Stein, "Partial differential equations" by L.C. Evans, " Lectures on nonlinear wave equations" by C. Sogge.

2004 Lecturers:

Undergraduate Lecturers

Cristina Pereyra, University of New Mexico
Lesley Ward, Harvey Mudd College

Graduate Lecturers

Jill Pipher, Brown University
Gigliola Staffilani, Massachusetts Institute of Technology

2004 Overall Program Schedule:

Monday, May 17

12:00-5:00 p.m.	Registration (Fuld Hall Common Room)
3:00-4:30 p.m.	Tea (Fuld Hall Common Room)
4:00 p.m.	Tour of Institute campus (meet in Fuld Hall Common Room)
5:00 p.m.	Tour of Institute campus (meet in Fuld Hall Common Room)
6:00 p.m.	Informal supper and orientation, poster construction
	(Coffee Lounge, Institute Dining Hall)

Tuesday, May 18 - Friday, May 21 & Monday, May 24, Wednesday, May 26 and Thursday, May 27

8:00-10:30 a.m.	Breakfast (Dining Hall)
9:30 a.m.	Beginning Lecture Course
	(May 18, 24-25 in Bloomberg Lecture Hall)
	(May 19-21, 26-27 in Simonyi Hall Seminar Room)
10:30 a.m.	Break
10:45 a.m.	Advanced Lecture Course

	(May 18 & 24 in Bloomberg Lecture Hall)
	(May 19-21, 26-27 in Simonyi Hall Seminar Room)
12:00 noon	Lunch (Institute Dining Hall)
1:15 p.m.	Review Sessions (Beginning Lecture Course)
	Simonyi Hall Seminar Room
1:15 p.m.	Research Seminars
	West Building Lecture Hall
2:30 p.m.	Review Sessions (Advanced Lecture Course)
	West Building Lecture Hall
2:30 p.m.	Research Seminars
	Simonyi Hall Seminar Room
3:45 p.m.	Afternoon Tea (Fuld Hall Common Room)
4:00 p.m.	Colloquia (guest lectures)
5:15 p.m.	Women in Science Seminar (Dilworth Room)
7:00 p.m.	Informal group suppers arranged by participants

Tuesday, May 25	**Day at Princeton University**
9:00 a.m.	Beginning Lecture Course (Bloomberg Lecture Hall)
10:00 a.m.	Depart for Princeton University (lectures, panel, lunch, & dinner)

Saturday, May 22	**Reunion Activities**
8:30 a.m.	Coffee and Pastries (Fuld Hall Common Room)
9:30 a.m.	Advanced Lecture Course (Simonyi Hall Seminar Room)
10:45 a.m.	Colloquium (Simonyi Hall Seminar Room)
	Anna Gilbert, AT&T Research
12:00 noon	Lunch (Institute Dining Hall)
1:30 p.m.	Research Poster Session (Dilworth Room)
3:00 p.m.	Panel Discussion (Institute Dining Hall)
4:30 p.m.	Research Poster Session/Break (Dilworth Room)
6:30 p.m.	Pizza Party (Institute Dining Hall)

Friday, May 28	
8:00-10:30 a.m.	Breakfast (Dining Hall)
9:30 a.m.	Beginning Lecture Course (Simonyi Hall Seminar Room)
10:30 a.m.	Break
10:45 a.m.	Advanced Lecture Course (Simonyi Hall Seminar Room)
12:00 noon	Lunch
	Close of Program