

Lie Groups, Representations and Discrete Mathematics

Thursday, September 1, 2005 (All day) - Monday, May 1, 2006 (All day)
(2005-2006)

In recent years new and important connections have emerged between discrete subgroups of Lie groups, automorphic forms and arithmetic on the one hand, and questions in discrete mathematics, combinatorics, and graph theory on the other. One of the first examples of this interaction was the explicit construction of expanders (regular graphs with a high degree of connectedness) via Kazhdan's property T or via Selberg's theorem (λ_1 is greater than $3/16$). Some other important fruits of this interaction were the construction of Ramanujan graphs, using the Jacquet-Langlands correspondence and Deligne's theorem on Hecke eigenvalues (Ramanujan conjecture), construction of new finitely presented simple groups via ergodic theory of lattices (à la Margulis) in a product of two trees, and a conceptual approach to the Product Replacement Algorithm of computational group theory (the convergence properties of the algorithm are related to the question of whether automorphism groups of free groups have property \tilde{T}). In each of these cases, an unexpected application to discrete mathematics was found by using ideas from number theory, Lie groups, representation theory and ergodic theory. Applications flow in the reverse direction as well.

A new combinatorial construction of expander graphs was used recently to resolve a group theoretic question on expansion in Cayley graphs. One reason these connections have been slow to emerge is that the fields involved are quite far apart, at least from the traditional viewpoint. People working in one side of the Lie Group/Discrete Math dichotomy are often not aware of the relevance of their work to the other side. Furthermore, each field has its own language and conceptual framework, so there is often a formidable language barrier to communication.

The program will be led by Alexander Lubotzky of The Hebrew University of Jerusalem who will be in residence at the Institute for the academic year. The goal of the program is to bring together mathematicians from several of the above-mentioned areas in order to strengthen the ties between the fields and generate further collaborations.

Information about a workshop in a related topic, "Automorphic Forms, Group Theory and Graph Expansion" to be held in February 2004 at IPAM can be found at: