

## abstract

LIE GROUPS, REPRESENTATIONS AND DISCRETE MATH

Topic:

Speaker:

Affiliation:

Date:

Time/Room:

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Consider an affine building of type  $A_{n-1}$ , which is a simplicial complex of dimension  $n$ . For  $n=1$ , this is a tree, which we will require to be homogeneous.

Consider the space of complex valued functions on the vertices of the building, and then consider the algebra  $A$  of invariant, finitely-supported difference operators. Here invariant can usually be taken to mean invariant with respect to the group of automorphisms on the building. (But for  $n=2$ , it can happen that there are not enough automorphisms. In that case one must define invariant differently.)

The algebra  $A$  is commutative. In fact, it is isomorphic to a polynomial algebra of degree  $n$  over the complexes. As such, it has an algebraic spectrum corresponding to affine  $n$ -space.

Since the elements of  $A$  are finitely supported, they preserve the space of  $\ell^2$  functions. Viewed this way,  $A$  is a self-adjoint operator algebra, and may be completed to a  $C^*$ -algebra. One can identify concretely the spectrum of this operator algebra as a subset of the algebraic spectrum of  $A$ .

The talk will be strictly expository: most of what's to be discussed was published by Tamagawa in 1963, and the rest by MacDonald in 1968. Moreover, because I like to draw pictures, discussion will center on the cases  $n=1$  (trees) and  $n=2$ .

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