abstract

Mathematical Conversations
Topic:

Speaker:

Affiliation:

Date:

Time/Room:

We relate algebraic quantum mechanics (C*-algebras) to topos theory, so as to capture the essence of quantum logic and quantum spaces. Motivated by Bohr's idea that the empirical content of quantum physics is accessible only through classical physics, we show how a C*-algebra of observables A induces a topos T(A) in which the amalgamation of all of its commutative subalgebras comprises a single commutative C*-algebra. According to the constructive Gelfand duality theorem of Banaschewski and Mulvey, the latter has an internal spectrum S(A) in T(A), which in our approach plays the role of a quantum phase space of the system. Thus we associate a locale (which is the topos-theoretical notion of a space and which intrinsically carries the logical structure of a Heyting algebra) to a C*-algebra (which is the noncommutative notion of a space). In this setting, states on A become probability measures (more precisely, valuations) on S(A), and self-adjoint elements of A define continuous functions. Formulated in this way, the observable part of the quantum theory defined by A is essentially turned into a classical theory, internal to the topos T(A).