abstract

Analysis Seminar
Topic:

Speaker:

Affiliation:

Date:

Time/Room:

As is well known, Eulerian simulations with a very small spatial mesh using an explicit scheme also require very small time steps, because the latter must be smaller than the time required to travel accross the mesh at the maximum flow velocity (mesh sweeping condition, also frequently - but somewhat improperly - called CFL condition).

In Lagrangian coordinates, fluid particles are labelled by their initial positions. The standard way of writing the equations in Lagrangian coordinates makes them very unappealing for numerical simulation. Cauchy's 1827 Lagrangian formulation of the incompressible Euler equations involves only Lagrangian gradients of the Lagragian map that relates initial and current fluid particle positions. From this, one can derive simple recursion relations among the time-Taylor coefficients of the the Lagrangian map. Bounds for suitable Hoelder norms of these Taylor cofficients can then be obtained. Such bounds imply the analyticity in time - for at least a finite time - of fluid particle trajectories when the initial data have limited smoothness, a result closely connected with work of P. Serfati (1994, 1995) and A. Shnirelman (2012).

From a numerical point of view, the recursion relations make it very easy to generate time Taylor series for the Lagrangian map. Combined with a new way of generating numerically the inverse Lagrangian map, this allows a temporal updating in which the time step is only constrained to be small compared to the nonlinear time (roughly the inverse of the largest velocity gradient). This should sharply reduce the algorithmic complexity of doing high-resolution simulations for the Cauchy problem.

The method is fairly general and can be applied to both incompressible and compressible flow. Actually, the idea is an outgrowth of the Lagrangian perturbation method developed by

cosmologists in the 90s to tackle the compressible Euler-Poisson equations in connection with the dynamics of large-scale structures in the Universe.

Abstract based on ongoing work with T. Matsumoto, S.S. Ray, B. Villone and V. Zheligovsky