

# **abstract**

ARITHMETIC COMBINATORICS

Topic:

Speaker:

Affiliation:

Date:

Time/Room:

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In 1977 Szemerédi proved that any subset of the integers of positive density contains arbitrarily long arithmetic progression. A couple of years later Furstenberg gave an ergodic theoretic proof for Szemerédi's theorem. At around the same time Furstenberg and Sarkozy independently proved that any subset of the integers of positive density contains a perfect square difference, namely elements  $x, y$  with  $x - y = n^2$  for some positive integer  $n$ .

In 1995, Bergelson and Leibman proved, using ergodic theoretic methods, a vast generalization of both Szemerédi's theorem and the Furstenberg-Sarkozy theorem, establishing the existence of arbitrarily long polynomial progression in subsets of the integers of positive density.

The ergodic theoretic methods are limited, to this day, to handling sets of positive density. However, in 2004 Green and Tao proved that the question of finding arithmetic progressions in some special subsets of the integers of zero density - for example the prime numbers - can be reduced to that of finding arithmetic progressions in subsets of positive density. In recent work with T. Tao we show that one can make a similar reduction for polynomial progressions, thus establishing, through the Bergelson-Leibman theorem, the existence of arbitrarily long polynomial progressions in the prime numbers.