

## abstract

COMPUTER SCIENCE/DISCRETE MATH II

Topic:

Speaker:

Affiliation:

Date:

Time/Room:

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Sipser and Gács, and independently Lautemann, proved in '83 that probabilistic polynomial time is contained in the second level of the polynomial-time hierarchy, i.e.  $BPP$  is in  $\Sigma_2^P$ . This is essentially the only non-trivial upper bound that we have on the power of probabilistic computation.

The Sipser-Gács-Lautemann simulation incurs a quadratic blow-up in the running time (i.e.,  $BPTIME(t) \subseteq \Sigma_2^P TIME(t^2)$ ). In this talk we show that this quadratic blow-up is in fact necessary for black-box simulations, such as those of Sipser, Gács, and Lautemann.

To obtain this result, we prove a new circuit lower bound for computing approximate majority, i.e. computing the majority of a given bit-string whose fraction of 1's is bounded away from  $1/2$ : We show that small depth-3 circuits for approximate majority must have bottom fan-in  $\Omega(\log n)$ .

On the positive side, we show a quasilinear time simulation of probabilistic time at the third level of the polynomial-time hierarchy (i.e.  $BPTIME(t) \subseteq \Sigma_3^P TIME(t \text{ polylog } t)$ ). If time permits, we will discuss some applications of our results to proving lower bounds on space-bounded randomized machines.