abstract

LIE GROUPS, REPRESENTATIONS AND DISCRETE MATH
Topic:

Speaker:

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In joint work with Yehuda Shalom, we have proved Margulis' Normal Subgroup Theorem for any discrete subgroup Γ of the automorphism group of a locally finite A_2-tilde building, B, provided that the quotient of B by Γ is compact.

The conclusion of the Normal Subgroup Theorem for a center-free group like this one is that any normal subgroup is either trivial or of finite index. Margulis proved this for irreducible lattice subgroups of higher rank, center-free semisimple Lie groups and semisimple linear groups over p-fields. Our result covers the special case of Margulis' where the semisimple group is $PGL_3(F)$ for some p-field F.

Our result also covers the case where the A_2-building is _not_associated to any linear group, but nonetheless admits a group Γ as above. Such examples are constructed in the two papers of [Cartwright, Mantero, Steger, Zappa, 1993], and also in recent unpublished work of [Steger, Trojan]. Curiously, in all these cases the full automorphism group of the building is itself discrete. By the way, it is possible for the full automorphism group of a locally finite A_2-building to be trivial. This is why you can't just say that Γ is supposed to be a uniform lattice in the full automorphism group.

In addition, our result covers all uniform lattice subgroups of $\Aut(\FF_q((T))\ltimes PGL_3(\FF_q((T)))$. It isn't known if such lattices are necessarily commensurable with lattices in $PGL_3(\FF_q((T)))$.

The strategy of proof is also Margulis': prove that if N is a nontrivial normal subgroup, then Γ/N has Property (T) and is also amenable. For Property (T), a proof depending only

on the basic geometry of the building was already available. Our work has been to translate and modify the proof of amenability so that it depends only on the geometry of the building, not on the additional structure possessed by linear groups. Margulis' proof is truly amazing. I would suggest taking a look at Chapter 8 of Zimmer's book to get the outline. Because the original proof is so amazing, the conversion to purely geometric terms is interesting, apart from the new cases which are now covered.