2010 Women and Mathematics Course Descriptions

Beginning Lecture Course - Week 1

Title: Class Field Theory for the p-adic numbers.
Lecturer: Elena Mantovan, CalTech
Teaching Assistant: Laura Peskin, CalTech

Lecture 1: p-ADIC NUMBERS. We introduce the field of p-adic numbers $\mathbf{Q}_P$. We discuss p-adic valuation, absolute value, topology, completion. We prove Hensel's lemma.
Lecture 2: p-ADIC LOCAL FIELDS. We study finite extensions of $\mathbf{Q}_P$. We introduce the notions of unramified, totaly ramified, tamely ramified and wildly ramified extensions. We prove Krasner's lemma and some structure theorems.
Lecture 3: THE ABSOLUTE GALOIS GROUP OF $\mathbf{Q}_P$. We study the Galois group of an algebraic closure of $\mathbf{Q}_p$. We introduce the Weil group, the Inertia subgroup and higher ramifications subgroups.
Lecture 4: CLASS FIELD THEORY FOR $\mathbf{Q}_P$. We study the maximal abelian extensions of $\mathbf{Q}_P$ and its Galois group over $\mathbf{Q}_P$. We prove the local Kronecker-Weber Theorem.

Prerequisites: Some knowledge of Abstract Algebra and basic Galois Theory will be helpful.

References:
[1] Cassels, Local Fields.
[2] Serre, Local Fields.
[3] Neukirch, J. Algebraic Number theory.

Advanced Lecture Course - Week 1

Title: Lectures on p-adic Galois representations.
Lecturer: Ariane Mezard, Versailles University
Teaching Assistant:Ramla Abdellatif, Universite Paris-Sud 11
1. Modulo p Galois representations
Serre fundamental characters
Irreducible mod p representations
Classification of 2-dimensional representations of Gal$\overline{\{\mathbf{Q}_p}$}/F) over $\overline{\mathrm{F}}_p$
2. p-adic Hodge Theory
Some rings of period
φ-modules

Modules with φ and connection
Equivalence of categories
$G_{F\infty}$-representations
Crystalline representations

References:
C. Breuil, Representations of Galois and of GL2 in characteristic p, (2007)
http://www.ihes.fr/~breuil/publications.html
C. Breuil and L. Berger, Towards a p-adic Langlands program, (2004)
http://www.ihes.fr/~breuil/publications.html
O. Brinon and B. Conrad, CMI summer school notes on p-adic Hodge Theory (2009).
L. Berger, C. Breuil, P. Colmez (eds) Représentations p-adiques de groupes p-adiques I, Astérisque 319.
P. Colmez, Lecture notes http://people.math.jussieu.fr/~colmez/M2.html
J.-M. Fontaine (ed), Périodes p-adiques, Asterisque 223
J.-M. Fontaine, Repréesentations p-adiques des corps locaux I, The Grothendieck Festschrift, Voll II, Prog. Math. 87, Birkhauser 1990, 249-309.
M. Kisin, Crystalline representations and F-crystals -- Algebraic Geometry and Number Theory, Drinfeld 50th Birthday volume, 459-496.

Beginning Lecture Course - Week 2

Title: The p-adic tree and the smooth mod p representations of GL(2;$\mathbf{Q}_p$).
Lecturer : Rachel Ollivier, Versailles University
Teaching Assistant:Katherine Korner, Harvard University

Lectures 1,2 : Construction of the Bruhat-Tits building for PGL(2;$\mathbf{Q}_p$). It is a tree endowed with a
natural action of GL(2;$\mathbf{Q}_p$), whose "boundary" can be identified with the projective line over $\mathbf{Q}_p$. We read
some features of the p-adic group GL(2;$\mathbf{Q}_p$) on the tree (parahoric subgroups, Cartan decomposition...).
Lectures 3,4 : Smooth representations of GL(2;$\mathbf{Q}_p$). We define the smooth mod p representations of
GL(2;$\mathbf{Q}_p$) and read the irreducible ones on the tree. We might have time to introduce the notion of
homological coefficient systems on the tree which allows to give resolutions for the latter representations.

Prerequisites: Some knowledge of p-adic numbers (beginning lecture course week 1) and of represen-
tation theory of finite groups will be helpful.

References:
[1] Kenneth, S. Buildings, Springer-Verlag (1989).

[2] Colmez, P. Preprint 5. Representations de GL2($\mathbf{Q}_p$) et (φ,Γ)-modules (2007). Paragraph 2.
[3] Paskunas, V. Coefficient systems and supersingular representations of GL2(F). Mém. Soc. Math. Fr. No. 99, (2004).
[4] Schneider, P. ; Stuhler, U. Representation theory and sheaves on the Bruhat-Tits building. Publications Mathématiques de l'IHÉS, 85 (1997).
[5] Serre, J.-P. Arbres, amalgames, SL2. Astérisque, No. 46. Société Mathématique de France, Paris, (1977). Or its translation : Trees, Springer.
[6] Vignéras, M.-F. Representations modulo p of the p-adic group GL(2; F). Compos. Math. 140, no. 2, 333{358 (2004).

Advanced Lecture Course - Week 2

Title: Introduction to the p-adic Langlands program.
Lecturer : Marie-France Vignéras. Paris VII Denis Diderot University
Teaching Assistant: Ana Caraiani, Harvard University

Synopsis:
1 - We will identify the p-adic representations of the complicated Galois group Gal_p of the field $\mathbf{Q}_p$ of p-adic numbers to finitely generated (φ,Γ)-modules over a certain commutative p-adic ring F (Fontaine's theorem). This is the first step towards the Langlands correspondence with the p-adic representations of GL(2,$\mathbf{Q}_p$), because the data (φ,Γ, F) is intimately related to the so-called (by Jacquet) mirabolic subgroup of GL(2,$\mathbf{Q}_p$).
2 - We will discuss the Colmez's algebraic construction of (φ,Γ) -modules over F killed by a power of p starting from representations of the mirabolic group. The basic tool will be the p-adic analogue of the Poincaré disk: the p-adic tree, and the homology of GL(2,$\mathbf{Q}_p$)-equivariant coefficient systems on the p-adic tree.
3 - The finiteness property of the (φ,Γ)-module over F when the mirabolic representation extends to a finite length representation of GL(2,$\mathbf{Q}_p$) is a difficult and delicate point; we will present two methods to solve it, by an elementary computation on the p-adic tree or by a less elementary conceptual method.
It is highly recommended to follow the parallel beginning lecture course on the p-adic tree.

Prerequisites: Some basic knowledge of p-numbers, Galois theory, number theory, and representation theory of finite groups at the undergraduate level will be helpful.

References:
1 - L. Berger, C. Breuil, P. Colmez (eds), Reprsentations p-adiques de groupes p-adiques I, Astrisque
no. 319.
2 - Colmez Pierre : Prepublications 5, 7 and 8 on his web page (to appear probably in 2010).
2 - Schneider Peter and Vignéras Marie-France : Preprint 5 on my web page (to appear in 2010)
3 - Vignéras Marie-France : Preprint 6 on my web page.