THE DIVINE SPARK

To the memory of Vladimir Voevodsky[1]

OO: Vladimir Voevodsky's life was unusual, even compared with other outstanding mathematicians. Indeed, among the strongest mathematicians of the twentieth and twenty-first centuries, not many can be named who did not have a complete university background and had no teachers in the usual sense. But you, according to Voevodsky, were a person who introduced "big mathematics" to him, even though you never were his formal advisor. How did it happen?

GSh: It was as follows. We both (especially Volodya) were incompatible with the structures of the last decade of socialism. Volodya was once again expelled from Moscow State University and worked as a technical assistant in a certain training facility. Possibly he was attracted by the computers there. As for me, not being a *komsomol*[2] member, at that time I had no opportunity to have an ordinary teaching job (which had always attracted me), and so, instead of working in a University, I was running an "Experimental Math Club" in that facility for fifth-grade school children, my son among them. Although my main official working place was then a certain computer center, I never stopped doing true mathematics, and usually I was surrounded by some pieces of paper with formulas on them. Voevodsky once noticed them and was very interested, since in that period of his life he was absolutely isolated from the world of pure math. Soon he asked me to suggest a problem to him. Such a request from a technical person seemed unusual, but I formulated a question for him that was not terribly difficult. Even more unusual was that a detailed answer was provided by Volodya in a week, based on the results of a computer experiment efficiently implemented by him. This answer turned out to be non-trivial. I would assign a student project of that level the highest grade without the slightest hesitation. This small result, obtained in a week by a person with a background of just two years of mathematical education, was very strong. But, of course, it is incomparable with Volodya's further achievements, including our joint ones.

OO: You said that you were both incompatible with the existing structures. But now, after the long time, a common point of view exists, according to which the Soviet Union was a paradise for scientists, especially for theoreticians not related to the military. The working conditions were much more pleasant than in the West or in modern Russia.

GSh: Quite possibly that was so for some highly ranked scientists or for the small community of mathematicians at the Steklov Institute in Moscow, whose only obligation was to do mathematics -- to prove theorems. But if you take the best Soviet mathematical journals of those years, first of all "Functional analysis and its applications" and "Uspekhi Matematicheskikh Nauk", you can just look at the authors' affiliations in the footnotes. Almost all the authors worked in strange places and institutes that had no relation to mathematics. And many of us, including me, (during the soviet regime I have changed jobs three times), usually had obligations that had nothing to do with pure math.

Volodya claimed definitely that he would never take an uninteresting job. It seemed to me unrealistic, since almost all the other Soviet mathematicians worked wherever they could find a position. But it turned out to be exactly as he had claimed. Never in his life, in any country, did he do something that was uninteresting to him.

OO: How did he turn from a promising technical assistant into a world-famous star? You were a witness...

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1 "Troitsky variant", No. 249 (March 13, 2018), pp. 2-3

2 a political youth organization in the Soviet Union

Vladimir Voevodsky, photo from www.ias.edu

GSh: I witnessed the main transformation. It should be stressed that before graduating from any university (by the way, he never graduated from Moscow State University), Volodya published five papers, two of them in international journals. All of them were of a high level. I was astonished by one of them from a non-mathematical point of view, since he wrote it while working on a student construction brigade[3] in, as far as I remember, Sakhalin. Before departing for there, he shared preliminary vague ideas about the corresponding paper with me, and upon his return the paper was complete. In September he gave a talk about it in the Shafarevich seminar, the most serious Moscow algebro-geometric event. The paper was devoted to étale topology – a refined and complicated part of modern mathematics. Writing such a remarkable paper while working on a construction crew is a miracle. You have to be Voevodsky to be able to do it!

OO: George, in your talk at the conference devoted to the memory of Voevodsky[4] you compared him with two outstanding historical personalities – Alexander Grothendieck and Evariste Galois. Why?

GSh: Firstly, all three, each in his own manner, were incompatible with the existing structures and morals of the society. And Voevodsky was, perhaps, the least conflictual of them: Galois was regularly vexed at those who were around him, and Grothendieck abandoned the mathematical community at the zenith of his professional career. Secondly, all three had no prestigious formal education, while most strong mathematicians graduated from the best universities. Galois was not admitted to Ecole Polytechnique, and Voevodsky was expelled from the math faculty of Moscow State University. As for Grothendieck, his mathematical education started in a concentration camp. Thirdly – and this is, of course, the most important point – all three (we are completely sure about the two elder ones) contributed a tremendous wealth of ideas to mathematics, which were substantially ahead of their time, and which strongly influenced the language and concepts of future generations.

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3 Sometimes translated as *building group* from the Russian word *stroyotryad*, this was a specific form of semi-official summer employment, popular among young soviet intellectuals, mostly men. Typically they involved hard physical labor, not restricted by official regulations, but paid much more money than student stipends or salaries in the scientific institutes.

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That their ideas were ahead of their time is easily demonstrated. The texts of Galois were read only about twenty years after his death. Grothendieck's ideas concerning the foundations of algebraic geometry changed the style of thinking and the language of this field, but widespread recognition and distribution of his ideas did not occur until several years after their first publication. Voevodsky has realized rather vague ideas of Grothendieck, and it seems that they have a promising future.

OO: But you and Voevodsky, moreover, turned out to be related with Grothendieck professionally. Tell me about your participation in the Grothendieck program.

GSh: Here I'll have to say a few words about Grothendieck's departure from the traditional math community. At the end of the 1960's he abandoned it completely, after he learned that the Institute of Advanced Studies near Paris, where Grothendieck worked, was partly supported by the military. He considered it a betrayal by mathematicians of the ideals of pure science and moved to the university of his youth in Montpellier. There he had to work with students who knew absolutely nothing. For such students Grothendieck invented an elementary area of study, which was a first for him, and called the corresponding objects *dessins d'enfants*, since they were essentially systems of lines drawn on curved closed surfaces. The reason for this name is that any child can draw such lines. Approximately in the middle of the Montpellier period of Grothendieck's life it suddenly turned out that the soviet mathematician Gennady Belyi proved a theorem that related the naïve dessins d'enfants activities with the serious topics that had occupied Grothendieck before his leaving traditional mathematics. Grothendieck wrote vividly about his emotional reaction to the Belyi theorem in the informal text "Esquisse d'un programme", written in 1984. This text reached Moscow quickly, and at that time Volodya and I already had some experience working together. We knew each other's tastes and felt that this branch of mathematics was for us.

OO: But how did it happen, technically? How could you know what Grothendieck was doing while he was leading the life of a hermit?

GSh: We need here a few words about the contacts of Soviet mathematicians with the world of science. Despite the iron curtain, these contacts were extensive enough. Pretending to be tourists, many leading mathematicians visited the USSR, especially Moscow. There was gossip that Pierre Deligne was able to climb over the fence of Moscow State University – I have not seen it myself, but I heard several of Deligne's talks in the seminars of Gel'fand and Shafarevich, after each of which he was available for personal discussions. The best mathematical journals reached us with a delay of two or three months. As for Grothendieck's program, it was a rather funny story. The text existed as a photocopied manuscript and spread over Moscow more quickly than in other places. My explanation is that the Moscow intellectuals were accustomed to reading *samizdat*[5]. So we were never embarrassed by the unclear status of this document.

OO: So a complicated algebro-geometric text was spread as samizdat?

GSh: Essentially, of course, yes; it is not precisely an algebra-geometric text – it is rather about the hidden relations of algebraic geometry with other domains of mathematics. It was not published until 1993. And I have discussed it with several outstanding foreign mathematicians who were sure that it was a delirium of a crazy person. In any case, not serious mathematics. Nowadays the attitude to Grothendieck program is totally different. There are hundreds of publications with references to it.

OO: Now it is already a classical manuscript.

GSh: Yes, now it is so, and Moscow was one of the first places where a serious attitude toward the program was formed. And a paper by Volodya and me, *"Drawing curves over number fields",* was the first mathematical paper where some ideas of the program were presented to the community in traditional form, with precise formulations, definitions and proofs. It was published in the volume devoted to Grothendieck's 60th birthday.

We had exactly three days to complete this paper, because it was impossible to send it abroad in the usual way, but there was a happy occasion: a French mathematician, Sansuc, was about to leave Moscow. In order to send it to the West legally, it was necessary to do things about which the young mathematicians nowadays have no idea…

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Wikipedia: ***Samizdat*** *was a form of dissident activity across the Eastern bloc in which individuals reproduced forbidden and underground publications by hand and passed the documents from reader to reader.* An essential part of Russian literature was known by intellectuals via samizdat rather than via formal publications: Mandelstam, Ahmatova, Pasternak, Bulgakov, Solzhenitsyn, …

OO: To get the permission of the «first department[6]»?

GSh: Yes, we needed some «first department» to confirm that the formulas in the paper did not disclose any state secrets. Volodya had no «first department», while at my job the request would have caused bewilderment.

OO: But did they, de-facto?!

GSh: Possibly…. But, thank God, currently these secrets have nothing to do with the military.

Volodya and I wanted to send the paper to the volume and personally to Grothendieck as well, but we did not know what would happen to a letter sent from Moscow, not to Paris, but to who knew where -- at that time Grothendieck was already a hermit.

The problem was solved by Sansuc, who had said a funny thing: «Grothendieck will have no contact with me, a bourgeois mathematician, but he will answer you politely, for you are repressed Soviet mathematicians». It happened exactly so. In the letter we attached to the paper we asked Grothendieck whether we had developed his ideas faithfully. He answered with a remarkable short note, in which one passage was of special interest. After polite wishes for further success, he wrote: «Probably, you are waiting for some mathematical comments. But I won't give any, *because of more urgent matters in this short life»*. As for his *short life* it can be recalled that all this happened around his 60th birthday, while he lived for 93 years (Voevodsky lived for 51, and Galois lived for only 21). It may be that in his philosophy the remaining 33 years were negligible. But during this «short» period Grothendieck managed to do lots of things. In addition to his mathematical studies he tried to understand physics and wrote an extraordinarily interesting autobiography.

OO: What is the relation between Voevodsky's work in the Grothendieck program and his greatest contribution to mathematics?

GSh: This relation is, perhaps, somewhat indirect.

The mathematical heritage of Grothendieck, from a superficial point of view, looks like it consists of the two disjoint parts. When he worked as a traditional mathematician and created the foundations of algebraic geometry, he was the head of a team producing new concepts and theorems, with many people involved, including mathematicians of world-wide renown. His *Esquisse d'un programme* is, on the other hand, a result of exotic reflections of a single individual (who was, however, assisted by two or three not so well known mathematicians).

The two parts have a common base, which I have started recently to call *extension of the topological intuition beyond the visible.*

Even as recently as the nineteenth century, it was hard to formalize topological intuition. For instance, it was not easy to give a precise definition of a *hole*.

OO: What is intuition in mathematics? And what sort of intuition did Voevodsky have?

GSh: He appreciated the main ideas of his predecessors, with Grothendieck and, perhaps, Milnor being the most important ones. He felt that a fundamental and well-motivated problem-solving strategy is to build a general theory. Hence, he said, it should be built – no matter how difficult it is. And maybe one of the main components of the Voevodsky intuition was the non-motivated conviction: in the *correct* theory *everything will be achieved.* This intuition led him to his best results.

Perhaps I did not mention that the particular results for which he earned the Fields medal are relatively easy applications of the general theory built by him, and this theory, in my opinion, is much more important than its applications. However, Volodya's particular results are wonderful and doubtless deserve the Fields medal.

OO: Why did Vladimir Voevodsky, after having obtained these results, soon become interested in other things? He was passionately involved with applying mathematical methods to population genetics. And then his final program followed, the univalent foundations of mathematics, where he tried to develop certain methods of verification of mathematical proofs. Why was his field [motivic homotopy theory] too narrow for him?

GSh: It happens regularly that winners of the Fields medal change their fields of activity after getting the prize. It happened with Novikov, Mumford, Grothendieck, and with Voevodsky, as well. One of the possible explanations: a person in the midst of life feels that, perhaps, he has already obtained

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Wikipedia: *The **First Department** (Russian: Первый отдел) was in charge of secrecy and political security of the workplace of every enterprise or institution of the Soviet Union that dealt with any kind of technical or scientific information.*

his best results. Afterward Volodya regarded his efforts in population genetics as fruitless, except that he succeeded a bit in the popularization of probability theory among algebraists.

As for the univalent foundations of mathematics, it is a remarkable direction, which, in my opinion, has great perspectives.

OO: And what is the humanitarian idea of this project? Is the idea to create the univalent foundations of mathematics important for the mankind?

GSh: This domain is relevant not only for the distant future of mankind. Specialists of various professions are currently active in it — mathematicians (especially logicians), programmers, and, in a smaller proportion, philosophers. One of Volodya's great achievements was gathering these people together for a special year in Princeton.

Volodya dreamed about more productive, than now, communication between humans and computers. Among humans it concerns mathematicians first of all, but other reasoning people as well.

Nowadays computers perform for us completely routine calculations, but, according to Volodya, the time has come when they should not only calculate, but also argue. Strictly speaking, this idea goes back to Leibniz rather than to Volodya, but it was first formulated in the centuries when it was just a fairy tale; however, nowadays this dream has a good chance to come true. Of course, intensive work on the automation of mathematical reasoning took place before Volodya as well, but his approach is unique in extending to this activity his homotopic intuition, that works beyond the traditional concepts not only of programmers, but even of logicians.

OO: How did mathematics change after Voevodsky?

GSh: As with Grothendieck, his heritage contains more traditional and less traditional parts.

The more traditional part contains his work in pure mathematics, some of which earned him the Fields medal. He made a tremendous contribution to the theory of motives, and some fantastic ideas of Grothendieck would have been buried and forgotten if Voevodsky had not understood and developed them. His impact on the theory of motives was so strong that currently it is one of the most active areas in abstract algebraic geometry, and this theory doubtless has a great future (it is my opinion, not Volodya's, who was generally skeptical about the future of pure math). The traditional part also contains less well known early papers (joint with me) by Volodya on Grothendieck dessins d'enfants — together with my students I have continued this activity for more than a quarter of a century, many mathematicians from various countries have joined it, and I also see the future of this theory.

The less traditional part is represented by the Voevodsky's papers and lectures on univalent foundations. The corresponding results are mostly incomplete and belong to a wider circle of problems than those of pure mathematics. In the large scale they belong to the general problem of *intelligence* — both to eternal aspects of these problems and to modern ones, related to the computerization of intellectual processes. The further development of Voevodsky's project needs the collective efforts of mathematicians and of people of other professions, and I hope that this project will not be abandoned after Volodya's passing away.

Voevodsky's entire life story confirms that *crazy ideas deserve attention.* They need just some sort of divine spark.

George Shabat
Interviewed by Olga Orlova